IV.8. Goursat's Theorem

Note. Recall that we have defined "function f is *analytic* on open set G if f is continuously differentiable at each point of G." We have shown that (Theorem IV.2.8) if f is analytic on $B(a; R)$ then f has a power series representation centered at a with radius of convergence $\geq R$.

Note. "Most modern books define an analytic function as one which is differentiable on an open set (not assuming the continuity of the derivative)" (page 100 of Conway). Goursat's Theorem shows that this approach to "analyticity" is equivalent to our's. First recall:

Theorem IV.5.10. Morera's Theorem.
Let G be a region (open, connected set) and let $f : G \to \mathbb{C}$ be a continuous function such that $\int_T f = 0$ for every triangular path T in G; then f is analytic in G.

Goursat's Theorem. Let G be an open set and let $f : G \to \mathbb{C}$ be a differentiable function; then f is analytic in G.

Revised: 8/6/2017