Chapter II. Metric Spaces and the Topology of $\mathbb{C}$
Study Guide

The following is a brief list of topics covered in Chapter II of Conway's *Functions of One Complex Variable*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section II.1. Definition and Examples of Metric Spaces.
Metric space, metric, open ball, closed ball, taxicab metric, Euclidean metric, metric versus norm, ℓ^2, ℓ^p for $1 \leq p < \infty$, Banach space, Hilbert space, open set, properties of open sets (Theorem 1.9), closed set, properties of closed sets (Theorem 1.11), topological space, interior of a set, closure of a set, boundary of a set, relationship between open/closed/interior/boundary and ε (Theorem 1.13), dense set.

Section II.2. Connectedness.
Connected metric space and connected sets, separation of a set, line segment, polygon, component of a set, components partition a set (Theorem 2.7), open set in $\mathbb{C}$ (Theorem 2.9).

Section II.3. Sequences and Completeness.
Convergent sequence, limit point of a set, closed sets and limit points (Propositions 3.2 and 3.4), Cauchy sequence, complete metric space, $\mathbb{C}$ is complete (Proposition 3.6), diameter of a set, "Cantor's Theorem."

Section II.4. Compactness.
compact set, open cover, a closed and bounded set in ℓ^2 that is not compact, compact sets are closed and bounded (Proposition 4.3), finite intersection property, every compact metric space is complete (Corollary 4.5), every infinite set in a compact metric space has a limit point (Corollary 4.6), sequentially compact metric space, Lebesgue's Covering Lemma, total boundedness, equivalences to compact (Proposition 4.9), the Heine-Borel Theorem (Theorem 4.10).

Section I.5. Continuity.
Continuous function at a point and on a set, equivalences to continuity (Propositions 5.2 and 5.3), limit of a function between topological spaces, definition of continuity for a function between topological spaces, uniformly continuous function, Lipschitz function, distance from a point to a set and properties (Proposition 5.7), properties of sets preserved by continuous functions (Theorem 5.8), Intermediate Value Theorem (Theorem 5.11), Extreme Value Theorem (Corollary 5.12), distance from a point to a compact set (Corollary 5.14), continuity on a compact set implies uniform continuity (Theorem 5.15), distance between two sets, distance from a closed to a compact set (Theorem 5.17).

Section II.6. Uniform Convergence.
Uniform convergence of a sequence of functions, the uniform limit of continuous functions is continuous (Theorem 6.11), Weierstrass M-Test (Theorem 6.2).

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