Chapter III. Elementary Properties and Examples of Analytic Functions

Study Guide

The following is a brief list of topics covered in Chapter III of Conway's *Functions of One Complex Variable*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Supplement. A Primer of Lipschitz Functions.
Example of an infinitely differentiable function of a real variable which does not have a power series representation, derivative of a function between metric spaces, Lipschitz function, Lipschitz constant, locally Lipschitz function, "differentiable implies locally Lipschitz" (Theorem 1), Theorem 2 (bounded f' if and only if Lipschitz on an interval), locally Lipschitz on a compact set implies Lipschitz (Theorem 3), Lipschitz implies uniformly continuous (Theorem 4), C^n functions of a real variable, C^∞ functions of a real variable, analytic functions of a real variable (those with power series representation), degree n Lipschitz (the class of such functions on $[0,1]$ is denoted $\text{Lip}^{(n+1}([0,1])$), the "chain" of functions of a real variable starting with C^0 and ending with analytic, the "chain" of functions of a complex variable starting with C^0 and ending with C^1.

Section III.1. Power Series.
Convergence and absolute convergence of a series of complex numbers, absolute convergence in $\mathbb{C}$ implies convergence, $\overline{\lim}\, a_n$ and $\underline{\lim}\, a_n$, power series in $\mathbb{C}$, geometric series, radius of convergence and Theorem III.1.3, the ratio test (Proposition III.1.4), the exponential function, sums and products of power series (Proposition III.1.6).

Section III.2. Analytic Functions.
Definition of differentiable, differentiable implies continuity (Proposition III.2.2), **definition** of *analytic*, Chain Rule, differentiation of a series and a_n in terms of $f'a$ (Proposition III.2.5), $f'(z) = 0$ implies f is constant (Proposition III.2.10), properties of e^z (page 4 of the in-class notes), definition of sine and cosine, periodic function, branch of the logarithm on an open connected set, the totality of branches of the logarithm on an open connected set (Proposition III.2.19), derivative of a branch of the logarithm is $1/z$ (Corollary III.2.21), principal branch of the logarithm, definition of z^b for $b \in \mathbb{C}$, region, Cauchy-Riemann equations, harmonic function, f is analytic if and only if Cauchy-Riemann equations are satisfied (Theorem III.2.29), harmonic conjugates and Theorem III.2.30.

Section III.3. Analytic Functions as Mappings, Mobius Transformations.

Path, smooth path, piecewise smooth path, angle between two paths, nonzero derivative implies angle preservation (Theorem III.3.4), conformal map, linear fractional transformation (or bilinear transformation), Möbius transformation, Möbius transformation on $\mathbb{C}_\infty$, translation/dilation/rotation/inversion and Proposition III.3.6, a Möbius transformation is uniquely determined by the action on any three given points in $\mathbb{C}_\infty$ (Lemma on page 6 of the in-class notes), cross ratio, preservation of cross ration by Möbius transformations (Proposition III.3.8), any three distinct points can be mapped to any three distinct points (Proposition III.3.9), a Möbius transformation takes circles/clines to circles/clines (Theorem III.3.14), points symmetric with respect to a circle/cline and the geometric interpretation, the Symmetry Principle (Theorem III.3.19), orientation of a circle/cline, left and right side of a circle/cline, the Orientation Principle (Theorem III.3.21).

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