Chapter IV. Complex Integration
Study Guide

The following is a brief list of topics covered in Chapter IV of Conway's *Functions of One Complex Variable*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

IV.1. Riemann-Stieltjes Integrals.
Bounded variation, total variation, calculation of total variation (Proposition IV.1.3), Riemann-Stieltjes integral, linearity (Proposition IV.1.7), computation of Riemann-Stieltjes integrals (Theorem IV.1.9), trace, rectifiable path, line integral, equivalent paths, curve, $\int_\gamma f\,|dz|$, Properties of integrals (Proposition IV.1.7), our Fundamental Theorem of Calculus (Theorem IV.1.18), closed curve.

Section IV.2. Power Series Representation of Analytic Functions.
Liebniz's Rule (Proposition IV.2.1), Proposition IV.2.6 and its use in introducing series, "analytic implies power series" (Theorem IV.2.8), evaluation of integrals using Corollary IV.2.13, bounds on derivatives (Theorem IV.2.14).

Section IV.3. Zeros of Analytic Functions.
Multiplicity of zeros, entire function, Liouville's Theorem (Theorem IV.3.4), Fundamental Theorem of Algebra (Theorem IV.3.5), equivalent conditions to a function being identically zero (Theorem IV.3.7), factoring an analytic function (Corollary IV.3.9), the Maximum Modulus Theorem (Theorem IV.3.11).

Section IV.4. The Index of a Closed Curve.
Winding number/index and Proposition IV.4.1, $n(\gamma; a)$ is constant on components of a set (Theorem IV.4.4).

Section IV.5. Cauchy's Theorem and Integral Formula.
Cauchy's Integral Formula First Version (Theorem IV.5.4), Cauchy's Integral Formula Second Version (Theorem IV.5.6), Cauchy's Theorem First Version (Theorem IV.5.7), use of Cauchy's Theorem and Theorem IV.5.8 to evaluate integrals, Morera's Theorem (Theorem IV.5.10).

Section IV.6. The Homotopic Version of Cauchy's Theorem and Simple Connectivity.
Homotopic closed curves, "fundamental group," convex and star shaped sets, homotopic to zero and Proposition IV.6.4, Cauchy's Theorem Second Version (Theorem IV.6.6), Cauchy's Theorem Third Version (Theorem IV.6.7), fixed-end-point homotopic, Independence of Path Theorem (Theorem IV.6.13), simply connected set, Cauchy's Theorem Fourth Version (Theorem IV.6.15), branch of $\log(f(z))$ (Corollary IV.6.17).

Section IV.7. Counting Zeros; The Open Mapping Theorem.
Winding numbers and integrals (Theorem IV.7.2 and Corollary IV.7.3), simple root of an equation, behavior of analytic f near a zero of multiplicity m ("The Stability Theorem for Orders of Zeros of Equations, Theorem IV.7.4), the Open Mapping Theorem (Theorem IV.7.5).

Section IV.8. Goursat's Theorem.
Goursat's Theorem (differentiable on an open set implies analytic).

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