**DIFFERENTIAL EQUATIONS FINAL**
**DUE 5:00 PM THURSDAY, MAY 7**

NAME _____ STUDENT NUMBER _____

Each problem is worth 10 points. Show all work. Be neat and use equal signs where applicable.

1. Show that $y = x$ is a solution of the Legendre equation of order one

$$(1 - x^2)y'' - 2xy' + 2y = 0, \qquad -1 < x < 1,$$

and find a second linearly independent solution.

2. Find an implicit solution to $y' = xy^3(1 + x^2)^{-1/2}$.

3. The Hermite equation is

$$y'' - 2xy' + \lambda y = 0, \qquad -\infty < x < \infty,$$

where $\lambda$ is a constant. This equation is important in many branches of mathematical physics; for example, in quantum mechanics the Hermite equation arises in the investigation of the Schrödinger equation for a harmonic oscillator. Find a series solution for this equation. Find the coefficients of $x$ up to $x^6$.

4. Find the general solution of $y'' + 3y' + 2y = \dfrac{1}{1 + e^{2x}}$.

5. Find the general solution of $(3x^2 + 2xy) + (x + y^2)y' = 0$. NOT SOLVABLE!

6. A person is riding in a motorboat, and the combined weight of the person, motor, boat, and equipment is 480 lb. The motor exerts a constant force of 16 lb on the boat in the direction of motion, whereas the resistance (in pounds) is numerically equal to the square of the velocity (in feet per second). Suppose the boat started from rest. Express the velocity as a function of time.

7. Find the general soultion of $y'' - 3y' - 4y = 3xe^{2x}$.

8. Solve the initial value problem $y''' - 2y'' + 4y' - 8y = 0$ with $y(0) = 2$, $y'(0) = 0$, and $y''(0) = 0$.

9. A 16 lb weight is attached to the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby strtching the spring 0.4 ft. Then, beginning at $t = 0$, an external force given by $F(t) = 40 \cos 16t$ is applied to the system. The medium offers a resistance in pounds numerically equal to $4x'$, where $x'$ is the instantaneous velocity in feet per second. Find the displacement of the weight as a function of time.

10. A circuit has in series an electromotive force given by $E(t) = 5 \sin 100t$ V, a resistor of 10 $\Omega$, and inductor of 0.05 H, and a capacitor of $2 \times 10^{-4}$ farads. If the inital current and the initial charge on the capacitor are both zero, find the charge on the capacitor at any time $t > 0$.