DIFFERENTIAL EQUATIONS TEST III

9:10-10:25

NAME _____ STUDENT NUMBER _____

Each problem is worth ~~20~~ 25 points. Show all work. Be neat and use equal signs where applicable. No calculators.

1. Sir George Airy (1801-1892) was an English astronomer and mathematician. He was director of the Greenwich Observatory from 1835 to 1881. *Airy's equation* is

$$y'' = xy.$$

One reason Airy's equation is of interest is that for x negative the solutions are oscillatory, similar to trigonometric functions, and for x positive they are monotonic increasing, similar to hyperbolic functions. Find a power series solution of this equation. Express the solution as a linear combination of two linearly independent functions. Find the coefficients of the powers of x up to x^7.

2. Use the Method of Frobenius to find a (series) solution of

$$2x^2y'' - xy' + (1+x)y = 0$$

for $x \in (0, s)$ for some "small" positive s. Find at least 6 coefficients of powers of x. Express the solution as a linear combination of two linearly independent functions. Also, make some comment about the singular point(s) and justify the comment.

3. A 16-lb weight is placed upon the lower end of a coil spring suspended vertically from a fixed support. The weight comes to rest in its equilibrium position, thereby stretching the spring 6 in. Determine the resulting displacement as a function of time if the weight is then pushed up 4 in. above its equilibrium position and released at $t = 0$ with an initial velocity of 2 ft/sec, directed downward. Don't worry about expressing the solution in terms of a single trigonometric function, as we often did in class. Also, notice that this is free, undamped motion.

4. A 16-lb weight is placed upon the lower end of a coil spring suspended from the ceiling and comes to rest in its equilibrium position, thereby stretching the spring 8 in. At time $t = 0$ the weight is then struck so as to set it into motion with an initial velocity of 2 ft/sec, directed downward. The medium offers a resistance in pounds numerically equal to $6x'$, where x' is the instantaneous velocity in feet per second. Determine the displacement of the weight as a funtion of time. Notice that this is free damped motion.