Section 1.2. Solutions

Note. In this section we give a few more definitions and discuss the forms of solutions.

Definition. Consider the nth order DE $F[x, y, y', y'', \dots, y^{(n)}] = 0$ where F is a real function of its $(n + 2)$ arguments $x, y, y', y'', \dots, y^{(n)}$.

1. Let F be a real function for all x in a real interval I and having an nth order derivative for all $x \in I$. The function f is called an *explicit solution* of the DE above on I if it fulfills the following: $F[x, y, y', y'', \dots, y^{(n)}(x)]$ is defined for all $x \in I$ and $F[x, y, y', y'', \dots, y^{(n)}(x)] = 0$ for all $x \in I$.

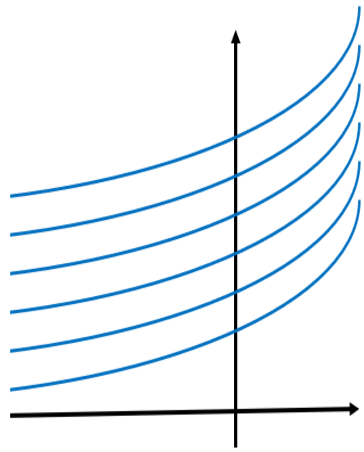
2. A relation $g(x, y) = 0$ is called an *implicit solution* of the above DE if this relation defines at least one real function f on I such that this function is a solution of the above DE.

Note. Function $f(x) = 2\sin x + 3\cos x$ is a solution of $y'' + y = 0$.

Example. Page 7, Example 1.7.

Note. Recall from calculus that when you find an indefinite integral you add an arbitrary constant (technically, an indefinite integral is a set of functions, any two of which differ by a constant). This means that certain DEs such as $y' = f(x)$ have solutions of the form $f = F(x, c)$ were c is the arbitrary constant (in fact, in this case c is simply added to an antiderivative of $f(x)$. There is said to be a *one parameter family of solutions* to the DE.

Example. If $y' = y$ then $y = e^x + c$ (or $y = 0$). This leads to the family of solutions (or *integral curves*):



Notice that if we had some additional piece of information about the solution y, we could eliminate the parameter c.

Note. It is rather rare that solutions to differential equations can be found in such a simple, explicit "closed form." More often, solutions appear as infinite series or as numerical approximations. In Chapters 2 and 4 we will deal with methods for finding explicit solutions to first and second order DEs. We will see applications of these in Chapters 3 and 5. Finally, in Chapter 6 we will obtains some solutions as series.

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