Theorem 3.10. Uniform Boundedness Principle.

If $X$ is complete, then a pointwise bounded subset $A$ of $B(X,Y)$ is bounded.

Proof (continued (again)). So for any sequence $(x^n)$ in $X$ we have bounded.

Theorem 3.11. Suppose that $(T_n)$ is a pointwise convergent sequence of bounded linear operators from a Banach space $X$ to a complete $Y$.

Then the limit $T(x) = \lim T_n(x)$ exists for each $x \in X$.

That is, $T \in L(X,Y)$ since $x$ is a unit vector. So $T \in L(X,Y)$.

Theorem 2.6. If $X$ is bounded. Since $X$ is a unit vector and $T$ is a continuous operator from $X$ to $Y$. By 

$\|T^n(x)\| \rightarrow 0$ and so $T$ is a bounded linear operator on $X$. So $T$ is bounded by $k$ for each $T \in A$. That is, $A$ is bounded.
So \( T \) and \( T' \) are bounded.

\[ \| x \| \leq \| y \| \quad \text{for all} \quad x, y \in X \]

\[ \| T(x) \| \leq \| T'(x) \| \quad \text{for all} \quad x \in X \]

\[ \| T(x) \| \leq \| T'(x) \| \quad \text{for all} \quad x \in X \]

**Proof (continued).** Then, if \( x \in X \) is a unit vector, we have

**Theorem 3.11.** Suppose that \( (T_n) \) is a pointwise convergent sequence of bounded linear operators from Banach space \( X \) to normed linear space \( Y \). Then, there exists a linear operator \( T \) such that \( T_n \to T \) in the norm topology.