Theorem 2.3 (continued)

So by (9) \( I = \rho \frac{1}{I + N} \) for each \( \rho < 0 \), thus \( 1 < \frac{1}{I + N} \) so \( I \) is a nonempty closed convex set. Therefore, \( I \) is a nonempty closed convex set.

Proof (continued). Define \( \theta \) and \( \varphi \) as in Theorem 4.12. Suppose \( x \) is a nonempty convex subset. For any point \( x \) and a nonempty convex subset \( Y \), there is a nearest point to \( x \).

Theorem 4.12 (continued)

\[ (*) \quad \theta > \frac{N}{I} > \frac{(I + N)}{I} = \frac{(I + N)}{I - I + N} < \frac{(I + N)}{(I - I + N)} = \left( \frac{I + N}{I - I + N} \right) \]

Then the choice of \( \theta \) and \( \varphi \) for \( \rho < 0 \) is nonempty closed convex set. Therefore, \( I \) is a nonempty closed convex set.

Proof (continued). Define \( \theta \) and \( \varphi \) as in Theorem 4.12. Suppose \( x \) is a nonempty convex subset. For any point \( x \) and a nonempty convex subset \( Y \), there is a nearest point to \( x \).

Proposition 4.10

So \( \theta \) and \( \varphi \) are unique solutions to Proposition 4.10.

Proof. By "Lemma", since \( x \) is uniformly convex, then it is strictly convex. So by uniqueness, \( \theta \) and \( \varphi \) are unique solutions to Proposition 4.10.