Theorem 9.18 (continued)

Proposition 9.17. If $M$ is invariant under $T$, then $T^*$ is self-adjoint on $M$. Moreover, if $M$ is invariant under $T$ and $T^*$ is self-adjoint on $M$, then $M$ is invariant under $T$.

Proof. Let $x, y \in M$ and for all $x, y \in M$, restricted to $M$ and $M^*$ is self-adjoint (that is, $\langle \lambda x, \lambda y \rangle = \lambda^* \langle x, y \rangle$). Since $T$ is self-adjoint on $M$ and $M^*$ are invariant under $T$, the $T^*$ is invariant under $T$. Since $x, y \in M$, therefore $\langle x, y \rangle = \langle T^* x, T^* y \rangle = \langle x, y \rangle$.

Proposition 9.17 (continued). If $M$ is invariant for compact self-adjoint operators $T$, then $T$ is invariant on $M$. Moreover, the restrictions of $T$ to both $M$ and $M^*$ are also self-adjoint.
Theorem 9.19 (continued 1)

Proof (continued). If \( T \) has an infinite number of eigenvalues, then by the spectral theorem for compact, self-adjoint operators (Theorem 9.18), the eigenspace for \( T \) is not the zero operator. By Proposition 9.14 (the spectral theorem for compact, self-adjoint operators (Theorem 9.18)), the eigenspace for \( T \) is an eigenspace of \( T \) with \( \lambda \neq 0 \), where \( \lambda \) is an eigenvalue of \( T \).

Theorem 9.19 (continued 2)

If \( \lambda \) is an eigenvalue of \( T \), then \( \lambda = \langle \lambda, \lambda \rangle \) by the definition of \( T \).

Proving that \( T \) is self-adjoint for all \( n \in N \). For \( x \in M \) where \( \langle x, x \rangle \neq 0 \), the eigenvector is \( x = x \). By Proposition 9.14 (the spectral theorem for compact, self-adjoint operators (Theorem 9.18)), the eigenspace for \( T \) is not the zero operator. By Proposition 9.14 (the spectral theorem for compact, self-adjoint operators (Theorem 9.18)), the eigenspace for \( T \) is an eigenspace of \( T \) with \( \lambda \neq 0 \), where \( \lambda \) is an eigenvalue of \( T \).

Theorem 9.19 (continued 2)

If \( \lambda \) is an eigenvalue of \( T \), then \( \lambda = \langle \lambda, \lambda \rangle \) by the definition of \( T \).
Theorem 9.20. A compact, self-adjoint operator $T$ on a separable Hilbert space is unitarily equivalent to a multiplication operator $M^g$ on $L^2$. Proof. Choose an orthonormal basis of eigenvectors $\{\phi_n\}$ and so is bijective. Now, $\phi$ is the norm of $f$. Then by Theorem 4.19 (see the descriptioen of $\phi$ above). That is, the $\phi$ corresponds but not to $\phi$. Such that $\phi = \phi$. Choose an orthonormal basis of eigenvectors $\{\phi_n\}$ and

Proof (continued). Since $T$ is compact, by Theorem 9.16, the nonzero elements of the form $\phi_n$ are eigenvectors. Since the spectral radius satisfies $R(T) > 0$, $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||$. Since $T$ is self-adjoint, then $R(T) = ||T||. $\Box$

Theorem 9.19 (continued 4)

Theorem 9.19 (continued 3)