Introduction to Functional Analysis, MATH 5740

Homework 5, Chapter 2

Due Tuesday, June 30 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

2.32. Prove that for $1 \leq q \leq p \leq \infty$, we have $\ell^q \subseteq \ell^p$.

2.41(a). Prove that (δ_n) is a Schauder basis for c_0 and for ℓ^p, $1 \leq p < \infty$. HINT: Show that the sequence of partial sums $\sum_{k=1}^{n} x(k)\delta_k$ converges to $x = (x(1), x(2), \ldots)$ with respect to the sup norm in c_0 and the ℓ^p norm in ℓ^p. Use an ε argument.

2.41(c). Prove that a space with a Schauder basis is separable. HINT: Let A be the set of all finite linear combinations of elements of the Schauder basis with rational coefficients (or rational complex coefficients). Then A is countable. For $x \in X$ and $\varepsilon > 0$, find $a \in A$ such that $a \in B(x; \varepsilon)$. Construct a by taking N sufficiently large such that $\| \sum_{k=1}^{N} \alpha_k b_k - x \| < \varepsilon/2$ (where $x = \sum_{k=1}^{\infty} \alpha_k b_k$ where $\{b_1, b_2, \ldots\}$ is the Schauder basis). Then approximate the first N parts of x with rational coefficients (within $\varepsilon/2$).

2.42. Prove that $T \in \mathcal{B}(X, Y)$ is a contraction if and only if $\|T\| < 1$.