Introduction to Functional Analysis, MATH 5740

Homework 7, Chapter 4

Due Wednesday, July 8 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

4.1. Prove that in any inner product space, two elements x and y are orthogonal if and only if $\|x + \alpha y\| = \|x - \alpha y\|$ for all $\alpha \in \mathbb{C}$.

4.2. Prove that ℓ^p for $1 \leq p \leq \infty$ is not an inner product space, except for $p = 2$. HINT: Use Theorem 4.8 to show that ℓ^2 is an inner product space by showing the ℓ^2 norm satisfies the Parallelogram Law. Show by example that the ℓ^p norm does not satisfy the Parallelogram Law for $p \in [0, \infty]$, $p \neq 2$.

4.4(a). Let (x_i) be an orthonormal set in a Hilbert space H. Prove Bessel's Inequality: For any $z \in H$, $\|z\|^2 \geq \sum_{i=1}^{\infty} |\langle z, x_i \rangle|^2$, and the equality holds for all z if and only if (x_n) is an orthonormal basis. HINT: Consider $\|z - \sum_{i=1}^{n} \langle z, x_i \rangle x_i\|^2$. When dealing with equality, use Theorem 4.17 and Theorem 4.14.