Introduction to Functional Analysis, MATH 5740

Homework 8, Chapter 4

Due Friday, July 10 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

4.16. Let $z_n = \frac{1}{\sqrt{2\pi}} e^{inx}$ for $n \in \mathbb{Z}$. Assume that (z_n) is an orthonormal basis for $L^2[-\pi, \pi]$. Consider $f(x) = x$ on $[-\pi, \pi]$. HINT: The Fundamental Theorem of Calculus holds for all of the integrals in this problem. Use Theorem 4.17(a) and (b).

(a) Calculate $\|f\|_2$.

(b) Calculate $\langle f, z_n \rangle$ for all $n \in \mathbb{Z}$.

(c) Bonus. Use the answers to Parts (a) and (b) to prove the identity $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.