Chapter 4. Hilbert Spaces

4.1. Introduction

Note. A Banach space is a complete normed linear space. A Hilbert space is a complete inner product space. Examples of linear spaces are $\mathbb{R}^n$ and $\mathbb{C}^n$. The Euclidean norm on these spaces is $\|(x_1, x_2, \ldots, x_n)\| = \sqrt{\sum_{k=1}^n |x_k|^2}$. An "inner product" is like the dot product on $\mathbb{R}^n$: $x \cdot y = \sum_{k=1}^n x_k y_k$. Recall that dot products are used to measure angles between vectors and, in particular, used to determine when vectors are orthogonal. Notice that the dot product on $\mathbb{R}^n$ induces the Euclidean norm on $\mathbb{R}^n$ as $\|x\| = \sqrt{x \cdot x}$. We have that all Hilbert spaces are examples of Banach spaces. Hilbert spaces have most of the geometric properties of $\mathbb{R}^n$.

Note. In this chapter, we assume that the field of scalars is $\mathbb{C}$.

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