

Chapter 3. Major Banach Space Theorems

Study Guide

The following is a brief list of topics covered in Chapter 3 of Promislow's *A First Course in Functional Analysis*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 3.1. Introduction.

Wiggle room.

Section 3.2. Baire Category Theorem.

Nested Set Theorem (Proposition 3.1), Baire's Theorem (Theorem 3.2), Dual Form of Baire's Theorem (Corollary 3.3).

Section 3.3. Open Mappings.

Properties of closures and linear transformations (Lemma 3.4), the Open Mapping Theorem (Theorem 3.5).

Section 3.4. Bounded Inverses.

Definition of a linear transformation is bounded below, invertibility of a linear transformation (Theorem 3.6), the Bounded Inverse Theorem.

Section 3.5. Closed Linear Operators.

Definition of the graph of a linear operator, closed linear operator, condition for T^{-1} to be closed (Theorem 3.7), the Closed Graph Theorem (Theorem 3.9).

Section 3.6. Uniform Boundedness Principle.

Pointwise bounded set of bounded linear operators, the Uniform Boundedness Principle (Theorem 3.10), the pointwise limit of a sequence of bounded linear operators between Banach spaces is bounded and linear (Theorem 3.11).

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