Chapter 5. Hahn-Banach Theorem Study Guide

The following is a brief list of topics covered in Chapter 5 of Promislow's A First Course in Functional Analysis. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 5.1. Introduction.

Motivation.

Section 5.2. Basic Version of Hahn-Banach Theorem.

Definition of Minkowski functional (or positively homogeneous subbadditive functional), Hahn-Banach Extension Theorem (Theorem 5.1).

Section 5.3. Complex Version of Hand-Banach Theorem.

The Complex Version of Hand-Banach Theorem (Theorem 5.3).

Section 5.4. Application to Normed Linear Spaces.

The Normed Linear Space Version of Hahn-Banach Extension Theorem (Theorem 5.4), linear functions which are 0 on a closed subspace and 1 at a point not in the subspace (Corollary 5.5), separating points with a bounded linear functional (Corollary 5.6), the norm of $x \in X$ in terms of the closed unit ball of X^* (Corollary 5.7).

Section 5.5. Geometric Versions of Hahn-Banach Theorem.

Linear manifold, hyperplane, hyperplanes as inverse images (Lemma 1), half spaces determined by hyperplanes, definition of separating sets with hyperplanes, internal points of a set, internal versus interior points (see Example 5.8), internal and interior points in a convex set in \mathbb{R}^n (Proposition 5.9), external points, bounding points, convex sets and Minkowski functionals (Proposition 5.11), properties of Minkowski functional p(x) (Proposition 5.12), the Minkowski functional p_K where K is a convex set (or "gauge functional"), Geometric Hahn-Banach Extension Theorem (theorem 5.14), Hahn-Banach Separation Theorem (Theorem 5.15), supporting hyperplane, strict separation of disjoint convex sets in a real linear space (Theorem 5.17), separation of convex sets in \mathbb{R}^n (Theorem 5.18).

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