Chapter 6. Duality
Study Guide

The following is a brief list of topics covered in Chapter 6 of Promislow's *A First Course in Functional Analysis*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 6.1. Examples of Dual Spaces.
Examples of duals given in Theorem 6.1, separability of X^* (Theorem 6.2), the Riesz Representation Theorem for the Dual of $L^p(E)$ (Theorem 6.3′).

Section 6.2. Adjoints.
Definition of adjoint, the difference in the definition of adjoint in a normed linear space and a Hilbert space, Example 6.5, dual basis, properties of the adjoint in the normed linear space setting (Theorem 6.6)

Section 6.3. Double Duals and Reflexivity.
Double dual, the linear isometry from X to X^{**} (Theorem 6.8), General Uniform Boundedness Principle (Theorem 6.9), reflexive Banach space, c_0 is nonreflexive, L^p is reflexive for $1 < p < \infty$ (Theorem 6.10), closed subspaces of reflexive Banach spaces are reflexive (Theorem 6.11), Banach space X is reflexive if and only if X^* is reflexive (Theorem 6.12).

Section 6.4. Weak and Weak* Convergence.
Weak convergence of a sequence normed linear space, convergence implies weak convergence (Lemma), weak convergence does not imply convergence (Example 6.13), weak limits are unique (Proposition 6.14), Continuity of Operations (Proposition 6.15), weakly convergent $\{f_n\}$ in $L^p(E)$ $(1 < p < \infty)$ converges if and only if $\lim \|f_n\| = \|f\|_p$ (The Radon-Riesz Theorem), in ℓ^1 weak convergence implies convergence (Proposition 6.16), weak* convergence of a sequence in a normed linear space, weak* convergence is weaker than weak convergence (Lemma).

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