Chapter 8. The Spectrum
Study Guide

The following is a brief list of topics covered in Chapter 8 of Promislow's *A First Course in Functional Analysis*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 8.1. Introduction.
Sift operators (Example 8.1), invertible bounded linear operator, spectrum $\sigma(T)$, the three types of values in the spectrum, point spectrum, continuous spectrum, residual spectrum.

Section 8.2. Banach Algebras.
Definition of an algebra, unit in an algebra, unital algebra, subalgebra, invertible element of a unital algebra, Products and Inverses Lemma, the spectrum in a unital algebra, Banach algebra.

Section 8.3. General Properties of the Spectrum.
The Spectral Mapping Theorem (Theorem 8.5), applications of the Spectral Mapping Theorem (Example 8.6), the spectrum of an inverse (Proposition 8.7), an invertibility condition on an element of a Banach algebra (Proposition 8.8), an element of a Banach algebra has a compact spectrum (Theorem 8.10), spectral radius, a bound for the spectral radius (Proposition 8.11), submultiplicative sequence of real numbers, quasi-nilpotent element of a Banach algebra, properties of the spectrum and spectral radius of elements of a Banach algebra (Theorems 8.15 and 8.16).

Section 8.4. Numerical Range.
Definition of numerical range and numerical radius for a bounded operator on a Hilbert space, the spectrum is contained in the numerical range (Proposition 8.17).

Section 8.5. Spectrum of a Normal Operator.
Properties of the spectrum of various operators on a Hilbert space (Proposition 8.18), properties of the spectrum for a self adjoint operator on a Hilbert space (Propositions 8.20 and 8.21, Theorem 8.23), properties of a normal operator (Proposition 8.22 and Theorem 8.23), orthogonality of eigenspaces for self adjoint operator (Proposition 8.24), properties of eigenvalues of a self adjoint operator (Proposition 8.25).

Section 8.6. Functions of Operators.
Functional calculus, properties of functions and polynomials of self adjoint operators on a Hilbert space (Proposition 8.26), self adjoint operators on finite dimensional spaces (Example 8.27).

Section 8.7. Brief Introduction to C^*-Algebras.
Definition of a C^*-algebra of bounded operators on a Hilbert space.

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