Chapter 9. Compact Operators
Study Guide

The following is a brief list of topics covered in Chapter 9 of Promislow's *A First Course in Functional Analysis*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 9.1. Introduction and Basic Definitions.
Relatively compact set, necessary and sufficient conditions for a set to be relatively compact (Lemma 9.1.A), compact operator, necessary and sufficient conditions for an operator to be compact (Lemma 9.1.B), T is compact if and only if $T(B(1))$ is relatively compact (Lemma 9.1.C).

Section 9.2. Compactness Criteria in Metric Spaces.
Definition of $A \subseteq^{\varepsilon} B$, totally bounded set and ε-net, totally bounded in terms of sequences (Proposition 9.1), relatively compact implies totally bounded (Corollary 9.2.A), in a complete space totally bounded implies relatively compact (Corollary 9.2.A), sufficient condition for a bounded set to be relatively compact (Proposition 9.2), equicontinuity, Arzela-Ascoli Theorem (Theorem 9.5), some relatively compact sets in ℓ^p (Theorem 9.6), compact multiplication operators on ℓ^p (Theorem 9.7), Example 9.8 as an example of a compact operator (Theorem 9.9).

Section 9.3. New Compact Operators from Old.
Compactness and Algebraic Properties (Proposition 9.10), Compactness and Limits (Proposition 9.11), Compactness and Adjoints (Proposition 9.12),

Section 9.4. Spectrum of a Compact Operator.
If T is compact then $S = T - \lambda I$ is onto if and only if it is one to one (Corollaries 9.4.A and 9.4.B), closed range (Proposition 9.15), properties of eigenvalues of a compact operator on a Banach space (Theorem 9.16),

Section 9.5. Compact Self Adjoint Operators on Hilbert Spaces.
Invariant set under an operator, invariance of $M^{\perp}$ and resulting self adjointness (Proposition 9.17), the Spectral Theorem for Compact Self Adjoint Operators (Theorem 9.18), expressing a compact self adjoint operator as a series of projections (Theorem 9.19), unilaterally equivalent, compact self adjoint operators on separable Hilbert spaces and multiplication operators on ℓ^2 (Theorem 9.20).

Section 9.6. Invariant Subspaces.
The Invariant Subspace Problem, solutions of the Invariant Subspace Problem by Enflo and Lomonosov, a bounded compact operator on a Banach space (of dimension greater than 1) has a closed proper invariant subspace (Theorem 9.21), invariant proper subspaces of a commuting operator (Theorem 9.22), the current status of the Invariant Subspace Problem: Does a general bounded linear operator on a Hilbert space have a proper closed invariant subspace?

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