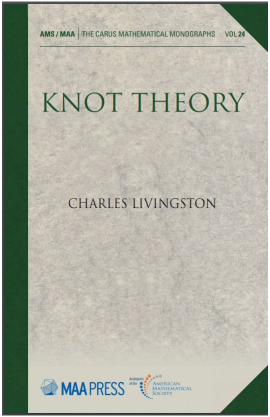
Introduction to Knot Theory

Chapter 6. Geometry, Algebra, and the Alexander Polynomial

6.3. The Signature of a Knot, and the other S-Equivalence Invariants—Proofs of Theorems



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Proof. Let V and W be Seifert matrices for knot K. Then by Theorem 6.2.3, V and W are S-equivalent. So we need to consider the two operations involved in the S-equivalence of matrices. The first operation (associated with bond moves in the Seifert surface) involves a matrix M, where $\det(M) = 1$, such that $W = MVM^t$. So $W = (MVM^t)^t = MV^tM^t$ and $(W + W^t) = MVM^t + MV^tM^t = M(V + V^t)M^t$. By Sylvesters Law of Inertia, the signatures of V and W are the same.

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It is to be shown that the manipulation of the Seifert matrix associated with stabilization does not affect the signature of $(V + V^t)$ in Exercise 6.3.5. □

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