Understanding the Concept of Division

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ABSTRACT

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The purpose of this study was to assess how well elementary students and mathematics educators understand the concept of division. Participants included 210 fourth and fifth grade students, 17 elementary math teachers, and seven collegiate level math faculty. The problems were designed to assess whether or not participants understood when a solution would include a remainder and if they could accurately explain their responses, including giving appropriate units to all numbers in their solution. The responses given by all participants and the methods used by the elementary students were analyzed. The results indicate that a significant number of the student participants had difficulties giving complete, accurate explanations to some of the problems. The results also indicate that both the elementary students and teachers had difficulties understanding when a solution will include a remainder and when it will include a fraction. The analysis of the methods used indicated that the use of long division or pictures produced the most accurate solutions.

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INTRODUCTION

Purpose

The purpose of this study was to assess the abilities of elementary students and mathematics educators in dealing with different types of division problems. We hypothesized that students may have some difficulties with understanding some of the concepts of division and this lack of understanding would be shown through their responses to the problems. The overall goal was to determine if students could correctly solve different types of division problems, regardless of the type of variable used, and give complete, accurate explanations of their answers. We hypothesized that teachers would also have difficulties with some of these concepts and that their abilities may be correlated with the abilities of their students. Finally, we hypothesized that the college mathematics faculty would demonstrate a greater level of understanding and give more accurate responses to the problems.

A secondary purpose of the study was to analyze the methods used by elementary students to solve division problems and determine the accuracy of each method. Our hypothesis was that the fifth grade students would primarily use long division, while the fourth grade students would use alternative methods, such as drawing pictures or using repeated subtraction. We believed that students that drew pictures would give more accurate responses than those that used the other methods. Additionally, we thought that students that were given more time to work in groups during class would be more successful and that the more closely a class followed the book, the less successful the students would be.

Literature Review

Teachers

Successfully teaching children to understand basic mathematical concepts is a difficult task. As a result, young students are often taught basic algorithms to solve certain types of problems with little emphasis on the underlying meanings behind how those algorithms work. Much research has concluded that part of the problem is that many elementary teachers do not understand some concepts well enough themselves in order to properly teach them (Ball, 1990a; Ball, 1990b; Goulding et al., 2002; Parmar, 2003; Simon, 1993). These studies stress that many teachers' understanding is focused on knowing rules and methods to solve problems and not on the fundamental concepts behind how a problem should be solved.

The concept of division is one that many students and teachers have problems understanding. Ball (1990a) found that prospective teachers are often unable to properly explain the underlying meaning behind division problems and cannot generate representations appropriate to some division problems. In this study, the prospective teachers struggled most with the meaning of division by fractions and many claimed to be uncomfortable doing calculations with fractions. Prospective teachers have also been found to have problems understanding the units in division problems. Teachers generally understand procedurally how to solve division problems, but many do not grasp which units correspond to which parts of the problem. Specifically, the prospective teachers often misunderstand the unit of the remainder or fractional part of the quotient in division problems (Simon 1993). This lack of understanding of units can play a large role in not fully understanding division.

In Ball's (1990b) study it was suggested that a possible reason for the lack of understanding found in prospective teachers is that their own understanding is not challenged when they are learning to teach. Because of this, they are not adequately prepared to challenge their students' understanding when they are teaching. It is commonly believed that since those with a degree in math have taken higher level math classes, they have greater conceptual understanding and are better prepared to teach than those without a mathematics degree. In Ball's study, it was found that those that had taken more collegiate level math courses did not have an advantage over those that had not taken such courses. Ball claims that this is because prospective teachers do not develop a deeper understanding of elementary level concepts such as division and fractions while taking college level math courses. Both groups of students, the elementary education and math majors, had difficulties validating their responses to the problems they were given. Goulding, et al. (2002) likewise found that whether or not a person has a degree in mathematics makes little difference in their effectiveness in teaching elementary mathematics.

Students

Children's development of math skills occurs far earlier in the context of the real world than it does in class. While they may not develop the ability to use algorithms before learning them in class, children are able to deal with basic arithmetic, including division and fractions, in their everyday life before they formally learn these concepts in school (Parmar, 2003; Mix et al., 1999). As early as four years old, children demonstrate an understanding of fractions and can solve problems that include fractions (Mix et al., 1999). When not in a school setting, it has been found that children use self-invented procedures to solve problems. These procedures, which are often more effective than the use of algorithms learned in school, are derived from real-life situations and demonstrate an understanding of how the problem should be solved. It has been suggested that teachers should encourage the use of such self-invented procedures in a classroom setting, as opposed to focusing solely on learning to use algorithms. Additionally, the math done in class should simulate real-life situations that the children can relate to (Verschaffel et al., 2006).

The concept of division is one that builds on previous knowledge of addition, subtraction, and multiplication. As with any such concept, it is important to build understanding early on in order to facilitate future learning (Parmar, 2003). For young children, a primitive model for division is repeated subtraction. Similarly, the primitive model for multiplication is repeated addition (Fischbein et al., 1985). These primitive models form a basis for future learning of these concepts. Following those models, it is easy to understand why many children hold misconceptions such as "multiplication makes bigger" and "division makes smaller" (Graeber & Tirosh, 1990). These misconceptions can play a large role in children's misunderstandings later on in their education, specifically when learning to do calculations with fraction and decimals.

Children's misunderstandings are frequently demonstrated through their inability to properly explain or interpret their responses (Silver et al., 1993; Cai & Silver, 1995; Graeber & Tirosh, 1990). Many students, when prompted to give a written explanation for their response to a problem, are unable to accurately do so. Similarly, it has been shown that students are often unable to write an appropriate word problem to accompany a given expression (Graeber & Tirosh, 1990). In Silver et al.'s (1993) study, it was found that while the students could correctly compute a given problem, they had difficulties in the "sense-making" phase of problem solving, which would enable them to accurately explain their solution. This study explains that asking children to give written explanations challenges their understanding of their response and that this should be a more common practice in classrooms.

Cai and Silver's (1995) study similarly concludes that U.S. students have difficulty giving correct explanations for their solutions to problems. This study investigates further into children's abilities to solve problems that include remainders. Knowing what to do with a remainder is a key element of making sense of a solution to a word problem. Depending on the units and wording of the problem, remainders may need to be expressed as a whole number, a fraction, or as a decimal. Often, students will deal with remainders by dropping them and giving a whole number answer, rounding their answer to the nearest whole number, or adding the remainder to their answer (Parmar, 2003). These problems may stem from the fact that students do not conceptually understand the method that they are using to solve division problems, so they are not adequately prepared to know what to do with a remainder when they have one.

It has been suggested that children's problems with fractions may be due partly to a misunderstanding of the symbols used to represent them (Mack, 1995; Mix et al., 1999; Verschaffel et al., 2006). Since the conventional symbol for a fraction, *a/b*, appears similar to whole numbers, students often misinterpret what the symbol means. While they may intuitively understand the concept of a fraction, students have problems translating the concept to the symbolic representation. Verschaffel et al. (2006) also suggests that students have confusion about the units that accompany a fraction. For example, students may not understand that *a/b* can refer to *a* out of *b* whole parts or *a/b* of a whole. Since students often relate the fraction symbol to their knowledge of whole numbers, many cannot grasp that *a/b* can refer to an amount that is less than a whole.

Math and Gender

Due to the vast difference in the number of males and females that are professional mathematicians, the issue of gender has always been prominent when discussing the subject of math (Ernest, 1976). Some believe that this difference is because males are inherently better at math, but while there may have been gender differences in the past, any significant advantage that males previously had over females in math has been diminishing with time. While overall ability is not significantly different, there are still some differences between the genders in terms of quantitative and spatial ability. Some research suggests that quantitative abilities develop at different times between the genders, giving females a slight advantage in their early years and males a slight advantage towards high school (Sadker et al., 1991). Some also suggest that differences in spatial abilities, which generally favor males, also play a role. However, the difference in spatial ability between the two genders has also been decreasing with time (Sadker et al., 1991; Ernest, 1976).

Ernest's (1976) study focuses on gender differences in elementary school children and the attitudes of their teachers. The results of the study indicate that almost half of the teachers in the study thought that boys did better in math than girls. After testing the performance of the students, no such difference was found. This study concludes that the idea that males are superior to females in mathematics is clearly a misconception, and that the lack of women that are professional mathematicians is likely due more to cultural influences than to lack of ability.

Word Problems

Word problems have always been a challenge for young students. When given such a problem, students are forced to figure out how the problem should be set up themselves, as opposed to being given numbers in a problem already set up for them. There are many factors that affect the complexity of word problems, especially when the problems include remainders, and these factors can cause students difficulty. Primarily, the complication stems from the fact that slight changes in wording can have a dramatic effect on the way in which the problem should be solved (Cai & Silver, 1995). An example taken from Cai and Silver (1995) is as follows:

> The computation 300/40 corresponds to each of the following problems:
>
> (a) If you have 300 candies to place into boxes that will each contain 40 candies, how many boxes will be filled with candy? (Answer: 7 boxes)
>
> (b) If you have 300 candies to place into boxes that will each obtain 40 candies, how many candies will be left over after you fill all the boxes that you can fill with candy? (Answer: 20 candies are left over)

(c) If you have 300 candies to place into boxes that will each contain 40 candies, how many boxes are needed to hold all of the candy? (Answer: 8 boxes)

In this example it is clear that slight variations in the wording can lead to dramatically different solutions to the problem, since each of these problems has a completely different answer, all coming from the computation 300/40.

Number of Variables

In this study, variables in word problems are being divided up into two distinct groups. The first group is made up of problems in which one variable is being divided into equal groups. In this type of problem there is only one possible unit for the answer, and if there is a remainder it is of the same unit as the quotient. This type of division is often termed partitive, which is defined in Fischbein et al. (1985) as an object or collection of objects being divided into equal subcollections. An example from Fischbein et al. of a partitive division problem is as follows:

In 8 boxes there are 96 bottles of mineral water. How many bottles are in each box? (Answer: 12 bottles)

In a problem of this type, one variable (bottles of mineral water) is being divided into groups (boxes). If there had been a remainder, it would also be in terms of bottles of water.

The second group of word problems is defined in this study as those that involve questions about more than one variable. When a problem is posed that has multiple variables it becomes quite a bit more complex to determine the complete solution to the problem. An example of this type of problem is as follows:

> You have 20 cups of flour and you want to make cakes. It takes 6 cups of flour to make a cake. How many cakes can you make from the flour that you have available? How much flour will be left over?

In this type of problem, it is necessary to understand that when you have completed the basic division step, you need to determine which unit corresponds to which part of your numerical solution. The first step in solving the problem is completing the basic division computation. Here, the computation 21/6 gives an answer of three with a remainder of two. It is then necessary to determine that the three corresponds to how many cakes you can make and the two corresponds to how much flour will be left over.

Types of Variables

The type of variable used in a problem determines whether or not the problem will have a remainder. In this study, both discrete and continuous variables were used in the problems. Discrete variables, such as people or books, cannot be divided up into pieces. Alternatively, continuous variables, such as length of fabric or amount of water, can be divided up into infinitely small pieces. An example of this distinction is as follows:

> (a) You have 19 books and you would like to give 3 people the same number of books. How many books will each person get? (Answer: 6 books with 1 book left over)
>
> (b) You have 19 ounces of water and you would like to give 3 people the same amount of water. How much water will each person get? (Answer: 6 1/3 ounces of water)

It is clear here that the use of a discrete versus a continuous variable changed the solution to the problem, which was otherwise identical.

Methods to Solve Division Problems

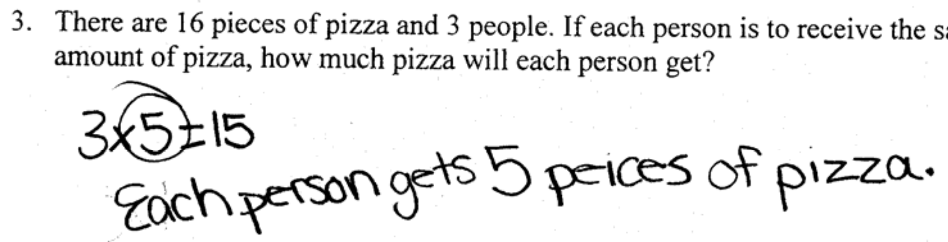
Long Division

Long division is the method used most often to solve basic division problems, especially when dealing with numbers of two digits or more. Using long division requires a good deal of understanding of multiplication and subtraction, and when used correctly, is a very accurate method of solving division problems. When using long division to solve a word problem, it is necessary to determine which variable given is the dividend and which is the divisor. (In a division problem, $a/b = c$, "a" is called the dividend, "b" is called the divisor, and "c" is called the quotient.) This step can be complicated by the many different ways that word problems are designed and is an important step in correctly solving the problem.

Multiplication

People will sometimes reverse the process and use multiplication to work backwards towards the solution. Using multiplication in this way demonstrates that the person understands the basic mathematical principle that multiplication is the inverse of division. Using this method requires knowledge of multiplication and can require the use of addition or subtraction if it is necessary to obtain a remainder. An example of a solution using multiplication is shown in Figure 1.1 below.

Figure 1.1: Sample Problem with Multiplication

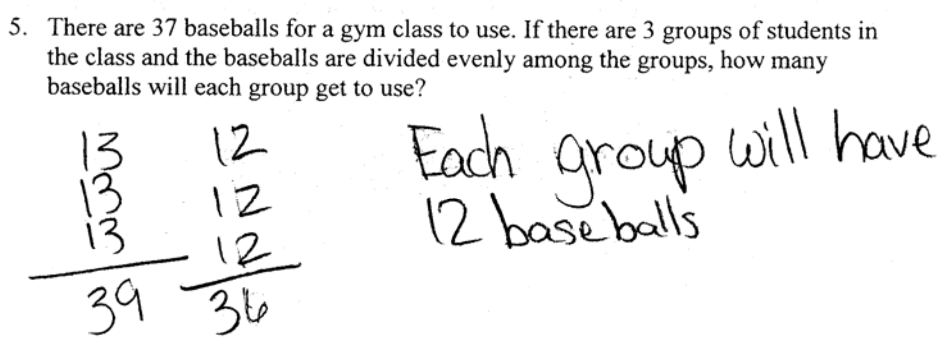

3. There are 16 pieces of pizza and 3 people. If each person is to receive the same amount of pizza, how much pizza will each person get?

3x5=15
Each person gets 5 peices of pizza.

Repeated Addition/Subtraction

A slightly more basic method that can be used to solve division problems is using repeated addition or subtraction. An important aspect of division is that it is essentially repeated subtraction, much like multiplication is repeated addition. The process of repeated subtraction involves beginning with the dividend and subtracting the divisor multiple times until the difference is zero or less than the divisor. Repeated addition involves repeatedly adding the divisor multiple times until the divided is reached. This process requires only knowledge of either addition or subtraction. An example of a solution to a word problem using repeated addition is shown below in Figure 1.2.

Figure 1.2: Sample Problem with Repeated Addition


5. There are 37 baseballs for a gym class to use. If there are 3 groups of students in the class and the baseballs are divided evenly among the groups, how many baseballs will each group get to use?

13 13 13 = 39
12 12 12 = 36

Each group will have 12 baseballs

Pictures

Often times drawing a picture to solve a problem, particularly a word problem, can make the process more clear and can help people to identify exactly what the problem is asking. Although this method does not require the use of division, using a picture to solve a division problem demonstrates a clear level of understanding of what is going on in the problem and can be extremely accurate.

METHODS

Participants

Participants were selected based on their position in one of three groups: a faculty member in the Mathematics Department at East Tennessee State University, an elementary teacher currently teaching mathematics to fourth or fifth grade students, or an elementary student currently enrolled in the fourth or fifth grade. Four elementary schools were included in the study, with a total of 210 students and 17 teachers. Seven ETSU Mathematics faculty members participated in the study.

Data Collection

The methods used to perform this research were approved by both the Institutional Review Board of East Tennessee State University and the Central Office of the Johnson City School System. All adult participants were given an informed consent document to read prior to their participation in the study. All children participants were required to have documented parental permission prior to their involvement and were read an additional consent document to obtain their assent.

Separate surveys were given to each of the members of the three groups. The children participants were given a survey consisting of a set of six math problems that dealt with different types of division and one question regarding their gender. The elementary teachers were given the same set of math problems and an additional set of questions regarding their age, gender, educational history, teaching history, and methods that they use to teach in their classroom. The ETSU Mathematics faculty members were given the set of questions pertaining to their age, gender, education, and teaching history as well as the set of math problems. (See Appendices E, F and G for specific questions for each group.)

All data for each group of elementary students and teachers was collected during regularly scheduled class time on site at each school and the data for the ETSU faculty was collected on the ETSU campus. The data collection took place over a period of five weeks. The children participants were given a maximum of thirty minutes to complete the survey and the adult participants were given a maximum of fifteen minutes to complete the survey. The instructions indicated that participants needed to show all of their work in the space given and write all of their answers in complete sentences. The students were read an additional set of instructions emphasizing the importance of showing work and giving complete explanations for answers. In addition, a solution with an explanation to a sample problem was verbally given to all student participants.

The Problems

The math problems in the surveys consisted of six word problems that dealt with different types of division. Each problem required division of a two digit number by a one digit number. There were three different types of problems with two problems of each type.

One Discrete Variable

Two of the problems involved one discrete variable being divided. Although there are two variables (people and groups, baseballs and groups) in each of the problems, a single variable is being divided into groups. These problems were intended to produce an answer with a whole number quotient and a whole number remainder, with the quotient and remainder of the same unit.

The two problems of this type are as follows:

2. There are 28 students in a class. If the class is divided into 5 groups with the same number of people in each group, how many people will be in each group?

5. There are 37 baseballs for a gym class to use. If there are 3 groups of students in the class and the baseballs are divided evenly among the groups, how many baseballs will each group get to use?

One Continuous Variable

Two of the problems involved one continuous variable being divided. These problems were intended to produce a quotient consisting of a whole number with a fraction and no remainder. The two problems of this type are as follows:

3. There are 16 pieces of pizza and 3 people. If each person is to receive the same amount of pizza, how much pizza will each person get?

6. You have 14 inches of rope. If the rope is shared evenly among 3 people, how much rope will each person get?

Two Variables

The final two problems involved questions about multiple variables. The solution to each of these problems includes a whole number quotient with a whole number remainder, where the remainder is of a different unit than the quotient. The problems of this type are as follows:

4. There are 27 yards of fabric. It takes 5 yards of fabric to make a dress. How many dresses can be made from the fabric? How much fabric will be left over?

7. There are 35 toys to be shared among a group of people. If each person is to receive 4 toys, how many people can the toys be shared with? How many toys will be left over?

Scoring

The surveys were analyzed using a standardized method of scoring. The problems were scored using several categories, with the first category for correct responses. To be correct a response had to have both the correct numbers and a correct explanation, including a unit with each number in the answer. The next category was for responses with correct numbers but with an error in the explanation. An answer with a wrong explanation, a partial explanation, or no explanation at all fell into this category. The third category was for problems in which the student encountered an error in their calculation. The students whose responses fell into this category had clearly begun the problem correctly but had made a calculation error, such as a multiplication or subtraction error during the long division process, which resulted in an incorrect final answer. The final group was for incorrect answers. The responses that fell into this category often did not show work and thus it was impossible to tell if they began correctly.

There were two additional categories for the problems that had a fraction in the correct solution. The first category was for responses that had the answer entirely correct, with the correct whole number, fraction, and explanation. The second additional category was for responses in which a fraction was given but was incorrect. The previous four categories remained the same but all pertained to responses given with a remainder.

Overall, a problem was considered correct for the four non-fraction problems if it fell into the first category listed above. Since students at this level have not directly been taught to use fractions with division problems, more responses were considered "correct" for the students for these problems. The students' responses were deemed correct if they had the correct whole number with the correct fraction or if they had the correct whole number portion of the answer. The adult participants were required to give a complete answer with a fraction in order for their response to be correct. For a detailed list of skills student of this age have learned regarding division and fractions see Appendix A. There were several possible correct responses for each problem, shown below in Table 2.1.

Table 2.1: Possible Correct Answers for each Problem

Problem	Possible Correct Answers
2. There are 28 students in a class. If the class is divided into 5 groups with the same number of people in each group, how many people will be in each group?	(a) 5 people in each group (b) 5 people in each group with 3 people left over (c) 3 groups of 6 people and 2 groups of 5 people (d) Impossible
3. There are 16 pieces of pizza and 3 people. If each person is to receive the same amount of pizza, how much pizza will each person get?	(a) 5 1/3 pieces of pizza (b) 5 pieces each with 1 piece left over (c) 5 pieces each
4. There are 27 yards of fabric. It takes 5 yards of fabric to make a dress. How many dresses can be made from the fabric? How much fabric will be left over?	(a) 5 dresses with 2 yards of fabric left over.
5. There are 37 baseballs for a gym class to use. If there are 3 groups of students in the class and the baseballs are divided evenly among the groups, how many baseballs will each group get to use?	(a) 12 baseballs in each group with 1 baseball left over (b) 12 baseballs in each group (c) 12 baseballs in 2 groups, 13 baseballs in 1 group
6. You have 14 inches of rope. If the rope is shared evenly among 3 people, how much rope will each person get?	(a) 4 2/3 inches of rope (b) 4 inches each, 2 inches left over (c) 4 inches each
7. There are 35 toys to be shared among a group of people. If each person is to receive 4 toys, how many people can the toys be shared with? How many toys will be left over?	(a) 8 people can share the toys with 3 toys left over

The methods used to obtain each response were also determined. Four primary methods were used to solve the division problems. These methods were long division, multiplication, repeated addition or subtraction, and pictures. A final category was made for students that did not show work. While most students used one method primarily to solve the problem, some used more than one method. In these cases, the method selected was what appeared to be the method from which their response came.

Statistical Analysis

All of the responses were counted for each participant and were organized by problem, grade, gender, and group. A group here is defined as all of the students taught math by the same teacher. For each participant, each response was treated independently in all analysis. The analysis of the responses included determining the total number of responses that fell into each category and the percentage of students that gave each response. The methods used by the students were counted and analyzed similarly. Analysis of the methods used also included determining the number of students that used each method that were successful in obtaining a correct answer. Comparisons were made between the various groupings in the analysis of both responses and methods. This information was then correlated with the information supplied by each teacher regarding their age, educational history, teaching experience, and methods used in their classroom.

RESULTS

Student Responses

One Discrete Variable

Overall, 291 of the 420 responses to problems two and five were correct, with 67.1 % of the responses to problem two correct and 71.4% of the responses to problem five correct. Tables 3.1 and 3.2 show the specific number of students from each grade and gender that gave each response. For each problem, slightly over 15% of the responses were incorrect, with most of the incorrect responses from the students in the fourth grade. The fewest number of responses to these problems were in the categories of explanation and calculation error.

Table 3.1: Responses to Problem 2

PROBLEM 2					
Group	**Correct**	**Explanation Error**	**Calculation Error**	**Wrong**	**Total Students**
4th grade females	23	3	6	13	45
4th grade males	34	5	1	10	50
4th grade – total	**57**	**8**	**7**	**23**	**95**
5th grade females	47	3	3	7	60
5th grade males	37	8	6	4	55
5th grade – total	**84**	**11**	**9**	**11**	**115**
All – total	**141**	**19**	**16**	**34**	**210**

Table 3.2: Responses to Problem 5

PROBLEM 5					
Group	**Correct**	**Explanation Error**	**Calculation Error**	**Wrong**	**Total Students**
4th grade females	25	2	3	15	45
4th grade males	31	1	2	16	50
4th grade - total	**56**	**3**	**5**	**31**	**95**
5th grade females	49	5	6	0	60
5th grade males	45	2	6	2	55
5th grade - total	**94**	**7**	**12**	**2**	**115**
All - total	**150**	**10**	**17**	**33**	**210**

One Continuous Variable

The number of correct responses with the remainder expressed as a fraction was extremely low for both problem three and problem six. Less than 7% of students gave the correct fraction for problem three and none of the students gave the correct fraction for problem six. An additional 5% gave an incorrect fraction for problem three and 2% gave an incorrect fraction for problem six. However, most students gave the correct whole number portion of the answer. Sixty-three percent of responses to problem three were correct without the fraction and almost 80% of the responses to problem six were correct without the fraction.

Most of the incorrect responses to these problems were from the fourth grade students and the percentage of responses with an explanation or calculation error was very low for both problems, similar to the previous two problems. The specific data for both problems is shown below in Tables 3.3 and 3.4.

Table 3.3: Responses to Problem 3

PROBLEM 3							
Group	**Correct with Fraction**	**Correct without Fraction**	**Explanation Error**	**Calculation Error**	**Wrong**	**Fraction Error**	**Total Students**
4th grade females	2	23	2	5	11	2	45
4th grade males	2	28	2	2	13	3	50
4th grade - total	**4**	**51**	**4**	**7**	**24**	**5**	**95**
5th grade females	6	42	3	3	4	2	60
5th grade males	4	40	3	0	5	3	55
5th grade - total	**10**	**82**	**6**	**3**	**9**	**5**	**115**
All - total	**14**	**133**	**10**	**10**	**33**	**10**	**210**

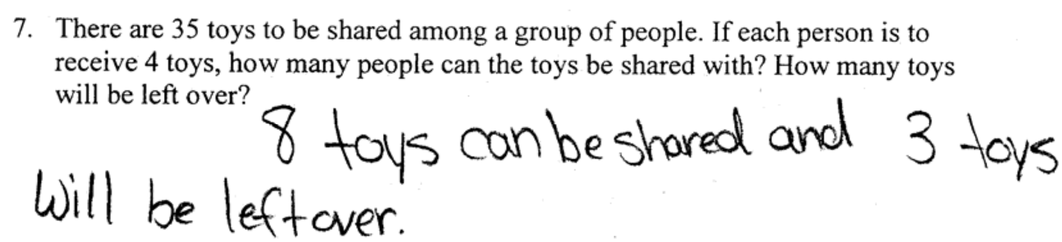
Table 3.4: Responses to Problem 6

PROBLEM 6							
Group	**Correct with Fraction**	**Correct without Fraction**	**Explanation Error**	**Calculation Error**	**Wrong**	**Fraction Error**	**Total Students**
4th grade females	0	32	3	2	7	1	45
4th grade males	0	36	2	2	9	1	50
4th grade - total	**0**	**68**	**5**	**4**	**16**	**2**	**95**
5th grade females	0	49	5	3	1	2	60
5th grade males	0	49	2	1	3	0	55
5th grade - total	**0**	**98**	**7**	**4**	**4**	**2**	**115**
All - total	**0**	**166**	**12**	**8**	**20**	**4**	**210**

Two Variables

The responses to problems four and seven were much differently distributed than the responses to the previous problems, with a much higher percentage of responses having an incorrect explanation. While most of the students gave the correct numerical response to these problems, almost 90% for problem four and almost 80% for problem seven, many did not give a correct explanation for their answer. Approximately 30% of the students' responses to both problems had an incorrect explanation of the numerical solution. An example of a solution that was given is shown below in Figure 3.1.

Figure 3.1: Sample Problem with Wrong Explanation


7. There are 35 toys to be shared among a group of people. If each person is to receive 4 toys, how many people can the toys be shared with? How many toys will be left over?

8 toys can be shared and 3 toys Will be left over.

Solutions of this type were common throughout both grades and genders, with many students using the same variable for both numbers in their response (as shown above), or leaving the unit off of one or both numbers altogether. Again, like the previous problems, very few students had errors in their calculations and most of the incorrect responses were from the fourth grade students. The specific data for these problems is shown below in Tables 3.5 and 3.6.

Table 3.5: Responses to Problem 4

PROBLEM 4					
Group	**Correct**	**Explanation Error**	**Calculation Error**	**Wrong**	**Total Students**
4th grade females	24	14	2	5	45
4th grade males	26	14	5	5	50
4th grade - total	**50**	**28**	**7**	**10**	**95**
5th grade females	42	13	4	1	60
5th grade males	34	19	0	2	55
5th grade - total	**76**	**32**	**4**	**3**	**115**
All - total	**126**	**60**	**11**	**13**	**210**

Table 3.6: Responses to Problem 7

PROBLEM 7					
Group	**Correct**	**Explanation Error**	**Calculation Error**	**Wrong**	**Total Students**
4th grade females	21	9	4	11	45
4th grade males	18	17	2	13	50
4th grade - total	**39**	**26**	**6**	**24**	**95**
5th grade females	30	20	6	4	60
5th grade males	32	17	3	3	55
5th grade - total	**62**	**37**	**9**	**7**	**115**
All - total	**101**	**63**	**15**	**31**	**210**

Overall Responses

Of the 1260 total responses given by the students, 841 responses were correct. This gives an overall percentage correct of 66.7% for all student participants. The overall percentage of correct responses by group, gender, grade, and problem is shown in Table 3.7. As shown, the fifth grade students had a much higher percentage of correct responses than the fourth grade students. The fourth graders gave almost four times the number of incorrect responses as the fifth graders, with a total of 128 incorrect responses from the fourth graders and only 36 incorrect responses from the fifth graders. The males and females had an almost identical percentage of correct responses. Between the groups there was a high amount of variability, ranging from 26.2% to 88.1% correct. There was also a moderate amount of variability between the percentages of correct responses by problem, with the lowest percentage for problem seven and the highest percentage for problem six.

Table 3.7: Overall Percentages by Group

Overall Percent Correct – Comparisons			
Group		**Grade**	
G4-1	65.7%	Grade 4	57.5%
G4-2	62.5%	Grade 5	74.3%
G4-3	38.1%	**Gender**	
G4-4	26.2%	Male	66.7%
G4-5	66.7%	Female	67.3%
G4-6	66.7%	**Problem**	
G4-7	60.0%	2	67.1%
G4-8	66.7%	3	73.3%
G4-9	58.3%	4	60.0%
G4-10	42.4%	5	71.4%
G5-1	75.5%	6	80.5%
G5-2	68.1%	7	48.1%
G5-3	88.1%		
G5-4	74.1%		
G5-5	71.4%		
G5-6	72.2%		
G5-7	70.4%		

Summary

Overall, a majority of the students gave correct responses to the division problems they were given. The percentage of correct responses was much higher for the problems that asked about only one variable: problems two, three, five, and six. While there was an equally high number of responses with the correct numerical answer to problems four and seven, the students had much more difficulty giving a correct explanation for their answers. The students did not tend to give a fractional response to the problems that involved the division of a continuous variable, with most giving a response that included a whole number remainder. While there was a lot of variability between the groups that participated in the study, overall the fifth graders gave a much higher percentage of correct responses than the fourth grade participants. There was little difference between the responses given by the males and those given by the females.

Methods

Overall

Most of the student participants used long division to solve the problems or did not show work at all. The next two most commonly used methods were pictures and multiplication, with the fewest number of students using repeated addition or subtraction. Table 3.8 shows both the number of students that used each method and the percentage of students that got the problem correct for each method. Long division and pictures had the highest percentage of correct responses. The method with the lowest percentage of correct responses was repeated addition and subtraction, with only one of the seven students that used this method giving a correct response.

Table 3.8: Methods

METHODS						
	Long Division			**Pictures**		
problem	**total**	**right**	**% right**	**total**	**right**	**% right**
2	104	86	82.7%	5	4	80.0%
3	99	81	81.8%	8	6	75.0%
4	91	59	64.8%	1	1	100.0%
5	112	85	75.9%	6	6	100.0%
6	94	84	89.4%	7	6	85.7%
7	101	56	55.4%	5	1	20.0%
total	**601**	**451**	**75.0%**	**32**	**24**	**75.0%**
% of total responses	**47.7%**			**2.5%**		

	Repeated +/-			**Multiplication**		
problem	**total**	**right**	**% right**	**total**	**right**	**% right**
2	0	0	n/a	5	2	40.0%
3	0	0	n/a	6	5	83.3%
4	1	0	0.0%	6	4	66.7%
5	1	1	100.0%	6	4	66.7%
6	3	0	0.0%	4	4	100.0%
7	2	0	0.0%	5	2	40.0%
total	**7**	**1**	**14.3%**	**32**	**21**	**65.6%**
% of total responses	**0.6%**			**2.5%**		

	No Work		
problem	**total**	**right**	**% right**
2	96	49	51.0%
3	97	62	63.9%
4	111	62	55.9%
5	85	54	63.5%
6	102	75	73.5%
7	97	42	43.3%
total	**588**	**344**	**58.5%**
% of total responses	**46.7%**		

Comparisons

The following figures show the comparisons between grade and gender for the methods used and percent correct for each method. Figure 3.2 shows that the fifth grade students used long division far more than the fourth grade students, and that the fourth grade students showed no work far more frequently than the fifth grade students. Figure 3.3 shows that overall the fifth graders had a higher percentage of correct responses when using any method to solve the problems. The exception to this is repeated addition or subtraction, which was not used by any fifth graders.

Figure 3.2: Methods Used – 4^{th} vs. 5^{th} Grade

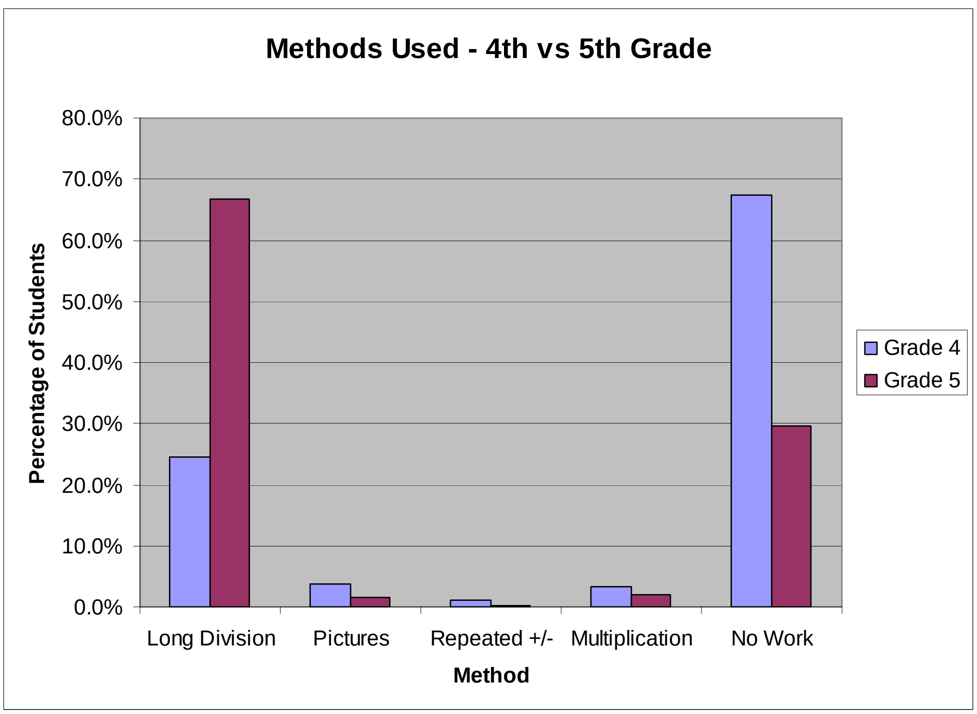
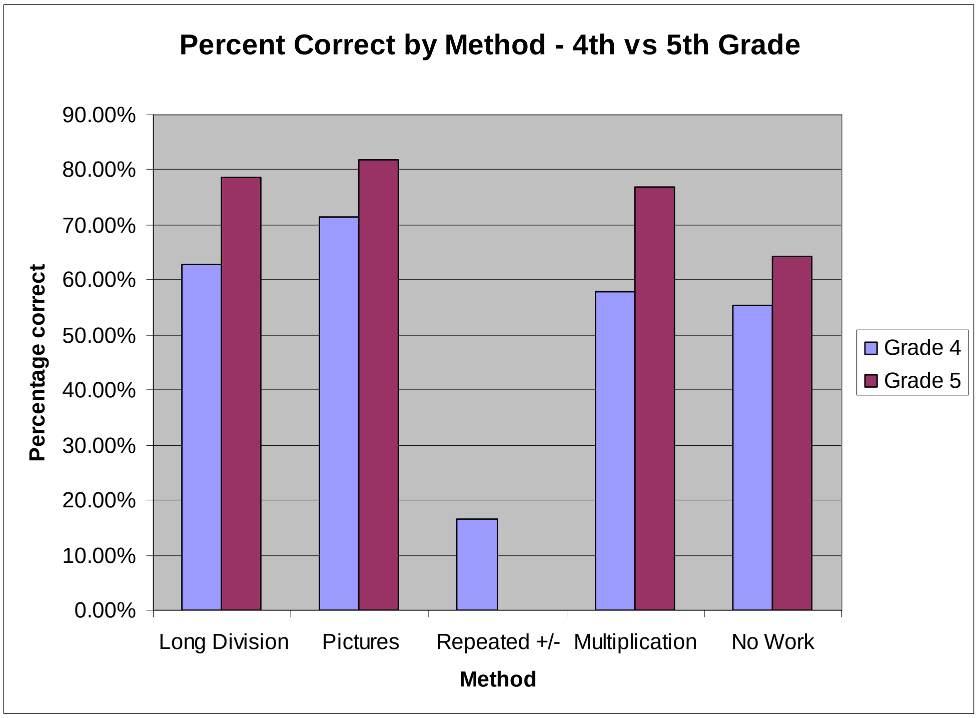


Figure 3.3: Percent Correct by Method – 4th vs. 5th Grade



Comparisons were also made between genders for the methods used. Figure 3.4 shows the differences in the percentage of male and female students that used each method. Females used long division slightly more often than males and males showed no work more often than females, but these differences were not significant. Figure 3.5 shows the percentage of correct responses given by males and females that used each method. The percentage of correct responses when using long division or when not showing work was nearly the same for males and females. The females gave a higher percentage of correct responses when using pictures to solve the problems and males were slightly more successful when using multiplication. Since there were no males that used repeated addition or subtraction, they do not have a percentage correct for this method.

Figure 3.4: Methods Used – Male vs. Female

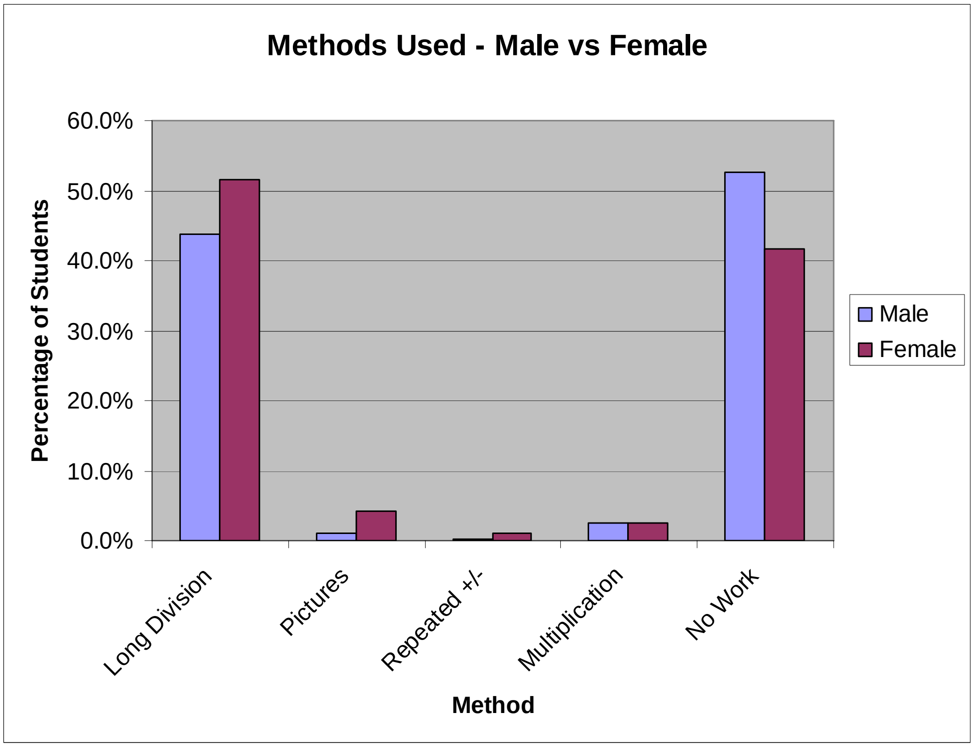
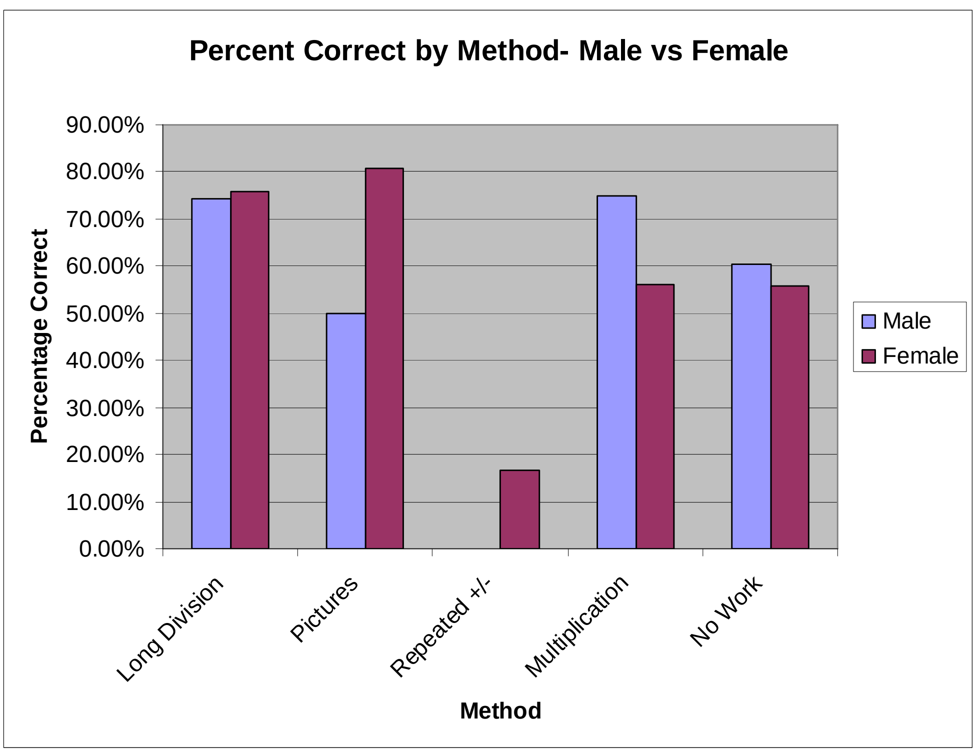


Figure 3.5: Percent Correct by Method – Male vs. Female



Summary

This data shows that long division was the method used most often to solve the division problems and that the other three methods were used quite infrequently, accounting for less than 6% of the total responses. Long division, pictures, and multiplication all had fairly high percentages of correct responses. Repeated addition or subtraction was used least frequently and had the lowest percentage of correct responses. For all methods that the fifth grade students used, they had a higher percentage of correct responses than the fourth grade students. This is in agreement with the results stated previously, which showed that the fifth graders overall had a higher percentage of correct responses than the fourth graders. While there were slight differences between the methods used by gender, none were significant. Similarly, with the exception of pictures, there were no large differences between the genders in terms of their percentage of correct responses when using the various methods.

Fractions

Methods

The following table shows the methods used by students that gave answers with fractions to the two problems that involved the division of a continuous variable. As shown in Table 3.9, most of the students that gave a fractional response showed no work. Long division was the most frequently used method to solve these problems. Although pictures were used by the fewest number of students, the highest percentage of correct responses came from students that used this method.

Table 3.9: Methods Used - Fractions

Methods Used- Fractions			
	total	correct	% correct
Long Division	10	5	50.0%
Pictures	3	2	66.7%
No Work	15	7	46.7%

A prime example of one of these problems that was solved using pictures is shown below in Figure 3.8. Figure 3.9 shows another sample problem in which long division was used.

Figure 3.6: Sample Problem – Fraction using Picture

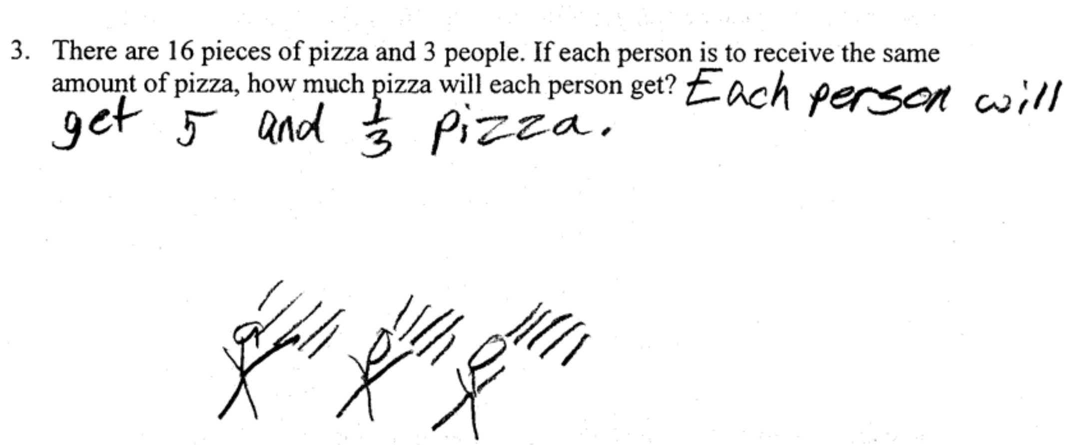
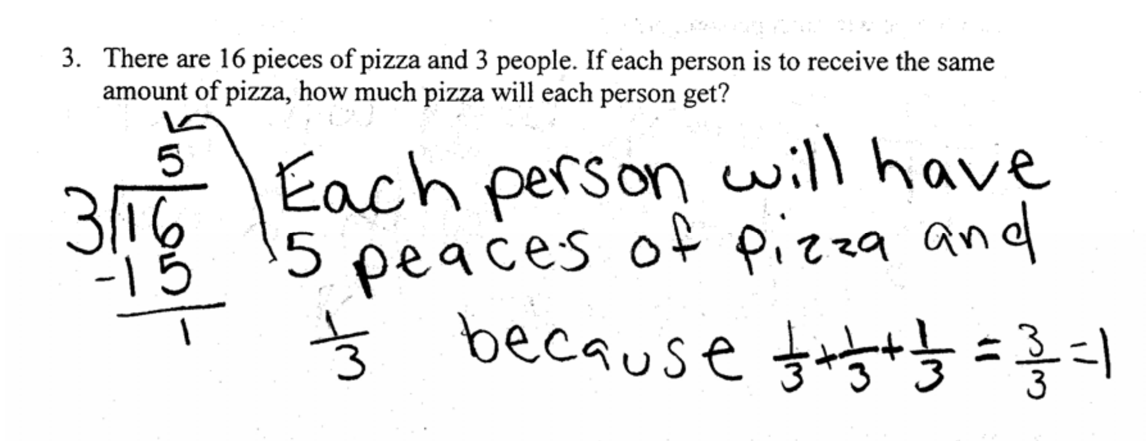


Figure 3.7: Sample Problem – Fraction using Long Division



Comparisons

Females gave a correct fraction slightly more often than males did, with 53% of the females' responses correct and 46% of the males' responses correct. There was a much larger difference between grades, with the percentage of correct fractions for the fourth graders at 36% and the fifth graders at almost 60%.

Summary

All of the results obtained from the analysis of the fraction responses are in keeping with the results from the rest of the study. The higher percentage of correct fraction responses came from the fifth graders, much like the earlier results, which show that the fifth graders to have an overall higher percentage of correct answers. Additionally, the difference between the number of correct fractions given by males and those given by females is relatively insignificant. In terms of the methods used, the three methods used here are the methods used most often overall. The highest percentage of correct responses came from the use of long division and pictures, much like the results from the overall responses.

Teacher Responses

The overall percentage of correct responses was slightly higher for the college faculty than the elementary teachers, with the college faculty having 85.7% correct responses and the elementary teachers at 75.5%. Tables 3.10 and 3.11 show the number of participants that gave each response. These tables show that the overall percentage correct for the fraction problems, problems three and six, were the lowest for both groups. The adults were required to give a fraction response to these problems in order for their response to be correct. A response given with the correct whole number portion of the answer but without the fraction was put in the "explanation error" category since the calculation was done correctly but a wrong final answer was given. The percentage correct of these problems was much higher for the college faculty than for the elementary teachers. Neither group had trouble explaining the units for the problems that asked about two different variables, problems four and seven.

Table 3.10: College Faculty Responses

COLLEGE FACULTY				
Problem	Correct	Explanation Error	Fraction Error	% Correct
2	6	1	0	85.7%
3	4	2	1	57.1%
4	7	0	0	100.0%
5	7	0	0	100.0%
6	5	2	0	71.4%
7	7	0	0	100.0%

Table 3.11: Elementary Teacher Responses

ELEMENTARY TEACHERS			
Problem	Correct	Explanation Error	% Correct
2	17	0	100.0%
3	5	12	31.3%
4	17	0	100.0%
5	17	0	100.0%
6	6	11	37.5%
7	15	2	87.5%

The following table gives the averages for the different questions that were asked to the adult participants. The education level was numbered using the following guidelines: 1 = Bachelor's Degree, 2 = Master's Degree, 3 = Doctoral Degree.

Table 3.12: Teacher Information Averages

Teacher Information Averages						
Group	Age	Education Level	Closeness to Book	Group Work	Total Years Teaching	Total Years Teaching at Current Level
Elementary Teachers	43.2	1.6	2.1	1.9	13.6	7.2
ETSU Faculty	53.6	3	n/a	n/a	27.0	25.6

Correlations

Correlations were done between this information for the elementary students and the overall percentage correct for each teacher's students. Table 3.13 shows the numbers and colors corresponding to each level of strength. Table 3.14 shows the absolute value correlations between the each teacher's information and the overall percentage of correct responses from their students. Since almost all of the elementary teachers were female, correlations were not done with gender. Correlations also were not done with the level of closeness to the book, since all but one teacher put two for their response. (To see the actual questions and the levels for "closeness to book" and "group work" refer to Appendix F.)

Table 3.13: Correlation Strengths

Correlations	
0.8 - 1.0	very strong
0.5 - 0.8	moderately strong
0.3 - 0.5	weak
0 - 0.3	very weak

Table 3.14: Correlations: Elementary Teacher Information with Student Success

Elementary Teacher Information with Overall % Correct of Students		
4th Grade Correlations (Abs. Value)		**Direction**
age	0.52	negative
total yrs teaching	0.47	negative
yrs at level	0.59	negative
degree	0.47	positive
group work	0.21	positive
5th Grade Correlations (Abs. Value)		**Direction**
age	0.58	positive
total yrs teaching	0.85	positive
yrs at level	0.92	positive
degree	0.42	negative
group work	0.27	negative

As shown, the correlations are extremely inconsistent between the grades. In the fourth grade, moderately strong negative correlations were found between the teacher's age and years they have been teaching with the success of their students. However, in the fifth grade, strong positive correlations were found between this information and the success of the students. In both groups there was a very weak correlation between the amount of time students spend working in groups and their success on the problems. Correlations were also determined between the percentage of correct responses from each teacher and their own information (age, education, etc.) and the percentage of correct responses from their students, but all were very weak.

Discussion

Since such a large percentage of students gave incorrect or incomplete explanations to some of the more complex questions, it is likely that some students do not fully comprehend the concept behind the algorithm they are using to solve division problems. It is also possible that the students, accustomed to not explaining their answers, simply did not try to give explanations. Many of the student participants, when instructed to give a complete explanation for their answer, responded that they did not know what that meant. Likewise, many of the teachers said that they did not require students to give explanations for their answers in class and that was not a skill they focused on.

It is also apparent that most of the students at this level do not understand when a problem will produce a remainder and when it will produce a fraction. While most probably intuitively understand that objects such as pizza and rope can be divided up into smaller pieces, they generally did not apply this concept when solving these problems. A possible reason for this is that a majority of these students had been taught to use long division to solve problems of this type. As such, it is probable that once the numbers in the problem had been put in the format of the long division algorithm, the students no longer paid attention to the wording of the problem or what type of variable was being used and how it might affect the solution. Since far fewer students attempted to give a fraction in their response to problem six than problem three, it is likely that 2/3 is a harder fraction for them to understand than ⅓. According to the Blueprint for Learning (see Appendix A,) children begin by learning the common fractions, ½, ⅓, and ¼. Therefore, children at this age may not be as comfortable with fractions with a numerator of two.

While the teachers did not have problems explaining their responses, a majority did not solve the problems with continuous variables correctly. Most of the elementary teachers, much like their students, gave an answer with a remainder for both of these problems. It is likely that they responded like this because this is the way they have been teaching their students to solve problems. As we expected, the college mathematics faculty gave the highest percentage of correct responses and solved the problems with fractions correctly more often than any other group.

In terms of methods used, our hypothesis that the fifth grade students would primarily use long division was supported. We expected the fourth grade students to use pictures and methods other than long division to solve the problems more often than the fifth grade students, which was somewhat true but not significantly so. We also believed that drawing pictures would produce more accurate responses than the other methods, while in reality long division and pictures produced almost the same percentage of correct responses. Additionally, we expected that students that were given more time to work in groups would be more successful on these problems, possibly because these students would have more time in class to creatively think of methods to solve problems than those that were not given time to work in groups. However, we did not find a significant correlation to support this. We also believed that the closer the teacher followed the book, the more constrained their students would be in terms of the methods they used and the less successful they would be. Since all but one teacher said that they followed the book "moderately" closely, we were unable to find a correlation between these variables.

In keeping with the literature, we did not find any significant differences between males and females. While there were some areas in which females had a slight advantage and visa versa, none were significant. We also did not find a significant correlation between the highest degrees obtained by the teachers with the success of their students, again supporting the literature, which suggested that higher levels of education did not prepare teachers to teach these basic concepts. Since the correlations drawn between the various information provided by the teachers and the success of the students were extremely inconsistent, no significant conclusions can be drawn.

Future Directions

To more fully assess the understanding of elementary math teachers and collegiate level math faculty, it would be beneficial to give them problems testing harder concepts, such as division by fractions. Giving them more challenging problems likely would have produced more conclusive results regarding their understanding. An additional change that could have been made would be to include sixth grade students in the study. Since sixth grade students have learned more about fractions and remainders, it would be interesting to give them the same set of problems to test how well they understand when they will have a remainder and when their answer will include a fraction. If one were to test this age group again, it would be beneficial to ask them more basic questions addressing concepts, such as pictorial problems with fractions that do not require a computation. This might enable us to more accurately pinpoint which areas are difficult for the students and which concepts they do not understand.

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APPENDICES

Appendix A: Blueprint for Learning

The information that follows was obtained from "A Blueprint for Learning: A Teacher's Guide to the Tennessee Curriculum." This guide is issued by the Tennessee Department of Education. This progression is a general overview and the exact information learned in each classroom thus far in the school year varies slightly between classes. The information included here is only what directly pertains to learning division and fractions and is not a complete list of skills that are learned in each grade.

Kindergarten

Determine if a figure has been divided into halves.

First Grade

Show ½ and ¼ or a set of objects.

Recognize one whole as two halves or four fourths.

Second Grade

Use concrete models or pictures to show whether a fraction is less than ½, more than ½, or equal to ½.

Compare the unit fractions ½, ⅓, and ¼.

Third Grade

Connect the spoken or written word names and concrete or pictorial representations (regions or sets) of fractions with denominators up to ten.

Connect written and pictorial representations of fractions with denominators up to ten.

Compare fractions with numerators of 1 and denominators up to 10.

Connect division to sharing situations.

Demonstrate multiplication using repeated addition (e.g., arrays).

Fourth Grade

Identify fractions as parts of whole units, as parts of sets, as locations on number lines, and as divisions of whole numbers.

Generate equivalent forms of whole numbers, commonly used fractions, decimals.

Represent numbers as both improper fractions and mixed numbers.

Use concrete or pictorial representations to compare and order commonly used fractions.

Use various models and equivalent forms to represent, order, and compare whole numbers and commonly used fractions and mixed numbers (e.g., number lines, base ten blocks, expanded notation, Venn diagrams, and hundreds boards).

Explain the relationship between multiplication and division.

Explain how addition, subtraction, multiplication, and division affect the size and order of numbers.

Add and subtract fractions with like denominators.

Divide efficiently and accurately with single-digit whole numbers.

Select the appropriate computational and operational method to solve word problems.

Solve story problems using whole numbers, fractions, and decimals (includes money).

Fifth Grade

Order and compare (<,>, or =) whole numbers, fractions, mixed numbers, and decimals using models (e.g., number lines, base ten blocks, Venn diagrams, and hundreds boards).

Compare and order fractions using the appropriate symbol (<,>, and =).

Represent proper fractions, improper fractions, and mixed numbers using concrete objects, pictures, and the number line.

Connect symbolic representations of proper and improper fractions to models of proper and improper fractions.

Represent numbers as both improper fractions and mixed numbers.

Identify and change improper fractions to mixed numbers and vice versa.

Generate equivalent forms of commonly used fractions, decimals, and percents (e.g., 1/10, ¼, ½, 0.75, 50%).

Recognize relationships among commonly used fractions and decimals.

Multiply a fraction by a multiple of its denominator (denominators less than or equal to 10).

Explain and demonstrate the inverse nature of multiplication and division.

Add, subtract, multiply, and divide whole numbers (multipliers and divisors no more than two digits).

Add and subtract commonly used fractions.

Solve real-world problems using decimals (including money), fractions, and percents.

Appendix B:

Copy of Permission from Central Office for JC Schools

Appendix C:

ETSU Institutional Review Board Approval

Appendix D:

Informed Consent Documentation

Adult Consent Document

Child Assent Document

Permission Letter for Parents of Children

Appendix E: Survey given to Elementary Students

Directions: Write all responses in complete sentences. Show all of your work.

1. Are you male or female? (Circle one)

2. There are 28 students in a class. If the class is divided into 5 groups with the same number of people in each group, how many people will be in each group?

3. There are 16 pieces of pizza and 3 people. If each person is to receive the same amount of pizza, how much pizza will each person get?

4. There are 27 yards of fabric. It takes 5 yards of fabric to make a dress. How many dresses can be made from the fabric? How much fabric will be left over?

5. There are 37 baseballs for a gym class to use. If there are 3 groups of students in the class and the baseballs are divided evenly among the groups, how many baseballs will each group get to use?

6. You have 14 inches of rope. If the rope is shared evenly among 3 people, how much rope will each person get?

7. There are 35 toys to be shared among a group of people. If each person is to receive 4 toys, how many people can the toys be shared with? How many toys will be left over?

Appendix F: Survey given to Elementary Math Teachers

Elementary Math Teachers

Directions: Please answer the following questions by writing in your response.

1. What is your age?
2. Are you male or female?
3. How long have you been teaching math?
4. How long have you been teaching math at this level?
5. List all degrees you have received.
6. From 1 to 3, how closely do you follow the book in your teaching? (1- not closely at all, 2- moderately, 3- follow exactly)
7. From 1 to 3, how often do you have your students work together in groups on math problems? (1- less than once per week, 2- once or twice per week, 3- three times per week or more)

The following section contains a set of word problems. Please show your work and write your responses in complete sentences.

1. There are 28 students in a class. If the class is divided into 5 groups with the same number of people in each group, how many people will be in each group?

2. There are 16 pieces of pizza and 3 people. If each person is to receive the same amount of pizza, how much pizza will each person get?

3. There are 27 yards of fabric. It takes 5 yards of fabric to make a dress. How many dresses can be made from the fabric? How much fabric will be left over?

4. There are 37 baseballs for a gym class to use. If there are 3 groups of students in the class and the baseballs are divided evenly among the groups, how many baseballs will each group get to use?

5. You have 14 inches of rope. If the rope is shared evenly among 3 people, how much rope will each person get?

6. There are 35 toys to be shared among a group of people. If each person is to receive 4 toys, how many people can the toys be shared with? How many toys will be left over?

Appendix G: Survey given to ETSU Mathematics Faculty

ETSU Math Faculty

Directions: Please answer the following questions by writing in your response.

1. What is your age?
2. Are you male or female?
3. How long have you been teaching math?
4. How long have you been teaching math at this level?
4. List all degrees you have received.

The following section contains a set of word problems. Please show your work and write your responses in complete sentences.

1. There are 28 students in a class. If the class is divided into 5 groups with the same number of people in each group, how many people will be in each group?

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