

Theorems, Conjectures, and Hypotheses Mentioned in *A Mathematical Mystery Tour*

Gödel's Incompleteness Theorem. Let T be a formal theory containing arithmetic. Then there is a sentence Φ which asserts its own unprovability and is such that: If T is consistent, $T \not\vdash \Phi$.

Fermat's Last Theorem. For no number $N > 3$ is there an integer solution to the equation

$$X^N + Y^N = Z^N.$$

Computer checks have verified this for all values of N up to 125,000.

The Goldbach Conjecture. Every even number is a sum of two prime numbers. Computer check has verified this for all even numbers up to 100 million.

The Riemann Hypothesis. The non-real roots of the Riemann zeta function $\zeta(z)$ all have real part $= 1/2$. Verified by computer to 2,000,000,000 zeros.

Classification Problem for 4-D Manifolds. Classify all possible 4-manifolds. This would give a description of all possible structures for the universe.

$P \neq NP$ Problem. Find a computational problem which can be solved on a computer in nondeterministic polynomial time but cannot be solved in polynomial time.

Invariant Subspace Problem for Hilbert Spaces. Does every bounded operator on a Hilbert space have a non-trivial closed, invariant space?

The Continuum Hypothesis. There is no set whose cardinal falls between \aleph_0 and 2^{\aleph_0} .