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Some generalizations of the Eneström-Kakeya theorem.

The authors give some generalizations of the Eneström theorem, which states that all
the zeros of a polynomial, with positive real coefficients $a_\nu$, lie in the annulus $\alpha \leq |z| \leq \beta$ when $\alpha = \min_{0 \leq \nu \leq n}\{a_\nu/a_{\nu+1}\}$ and $\beta = \max_{0 \leq \nu \leq n}\{a_\nu/a_{\nu+1}\}$. They prove Theorem
1, which states that all of the zeros of the polynomial $\sum_{\nu=0}^{n}(\alpha_\nu + i\beta_\nu)z^\nu$ lie in the
annulus $R_1 \leq |z| \leq R_2$, where for some $k$ and positive $t$, $\alpha_0 \leq t\alpha_1 \leq t^2\alpha_2 \leq \cdots \leq t^k\alpha_k \geq
\sum_{\nu=0}^{k+1}\alpha_{k+1} \geq \sum_{\nu=0}^{k+2}\alpha_{k+2} \geq \cdots \geq \sum_{\nu=0}^{n}\alpha_n$ and the expressions for $R_1$ and $R_2$ involve $\alpha_\nu$, $\beta_\nu$, $k$ and $t$. From this theorem the authors derive two corollaries and two more theorems. In their example, they show that all the zeros of $1 + 10z + 20z^2 + 40z^3 + 80z^4 + 50z^5$ lie in $0.499 \leq |z| \leq 0.840$.

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