A Bernstein type $L^p$ inequality for a certain class of polynomials. (English summary)


The classical theorem of S. Bernstein [Leçons sur les propriétés extrémales et la meilleure approximation des fonctions analytiques d’une variable réelle, Gauthier-Villars, Paris, 1926; JFM 52.0256.02] relates the supremum norm of a complex polynomial $P$ of degree at most $n$ and its derivative $P'$ by the Bernstein inequality $\sup_{|z|=1} |P'(z)| \leq n \cdot \sup_{|z|=1} |P(z)|$. The Bernstein inequality reduces to equality if and only if $P(z) = c z^n$ for some constant $c \in \mathbb{C}$. The inequality has been extended to the $L^p$ norm [A. Zygmund, Trigonometric series. 2nd ed. Vols. I, II, Cambridge Univ. Press, New York, 1959; MR0107776]. The paper under review provides an $L^p$-estimate ($0 \leq p \leq \infty$) of $P'$ in terms of $P$ for the class consisting of the complex polynomials $P$ of degree at most $n$ which are nonzero inside a sufficiently large neighborhood of the origin of the complex plane $\mathbb{C}$ and satisfy the condition $P'(0) = \cdots = P^{(m-1)}(0) = 0$ ($m \leq n$) at the origin of $\mathbb{C}$.

Walter Schempp

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