Reverse Directed Triple Systems

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ABSTRACT. A directed triple system of order \( v \) and index \( \lambda \), denoted \( DTS_\lambda(v) \), is said to be reverse if it admits an automorphism consisting of \( \frac{v}{2} \) transpositions when \( v \) is even, or a fixed point and \( \left( v - 1 \right)/2 \) transpositions when \( v \) is odd. We give necessary and sufficient conditions for the existence of a reverse \( DTS_\lambda(v) \) for all \( \lambda \geq 1 \).

1 Introduction

A directed triple system of order \( v \) and index \( \lambda \), denoted \( DTS_\lambda(v) \), is a \( v \)-element set \( X \), of points, together with a set \( B \), of ordered triples of elements of \( X \), called blocks, such that any ordered pair of points of \( X \) occurs in exactly \( \lambda \) blocks of \( B \). The notation \([x, y, z]\) will be used for the block containing the ordered pairs \((x, y)\), \((x, z)\), and \((y, z)\). Hung and Mendelsohn [6] introduced directed triple systems as a generalization of Steiner triple systems and showed that a \( DTS_1(v) \) exists if and only if \( v \equiv 0 \) or \( 1 \pmod{3} \). Seberry and Skillicorn [8] proved that a \( DTS_\lambda(v) \) exists if and only if \( \lambda v(v - 1) \equiv 0 \pmod{3} \), \( v \neq 2 \).

An automorphism of a \( DTS_\lambda(v) \) is a permutation of \( X \) which fixes \( B \). The orbit of a block under an automorphism \( \pi \) is the image of the block under the powers of \( \pi \). A collection of blocks \( \beta \) is said to be a collection

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of base blocks for a $DTS_{\lambda}(v)$ under the permutation $\pi$ if the orbits of the blocks of $\beta$ produce the $DTS_{\lambda}(v)$.

Several types of automorphisms have been explored in connection with the problem of determining the values $v$ for which there are certain types of block designs of order $v$ admitting the automorphism. In particular, a cyclic $DTS_{\lambda}(v)$ admits an automorphism consisting of a single cycle of length $v$ and exists if and only if $[2, 4]$:

1. $\lambda \equiv 0 \pmod{6}$ and $v \neq 2$, or
2. $\lambda \equiv 1$ or $5 \pmod{6}$ and $v \equiv 1, 4 \pmod{12}$, or
3. $\lambda \equiv 2$ or $4 \pmod{6}$ and $v \equiv 1 \pmod{3}$, or
4. $\lambda \equiv 3 \pmod{6}$ and $v \equiv 0, 1 \pmod{4}$.

A $DTS_{\lambda}(v)$ which admits an automorphism consisting of a fixed point and $k$ cycles of length $(v - 1)/k$ is said to be $k$-rotational. A $k$-rotational $DTS_{\lambda}(v)$ exists if and only if $\lambda v \equiv 0 \pmod{3}$ and $v \equiv 1 \pmod{k}$ [1]. A 1-rotational $DTS_{\lambda}(v)$ exists if and only if $\lambda v \equiv 0 \pmod{3}$ and $v \geq 3$ [3]. These two results, along with the observation that $\lambda kv \equiv 0 \pmod{3}$ is a necessary condition for the existence of a $k$-rotational $DTS_{\lambda}(v)$, yield:

**Corollary 1.1.** A $k$-rotational $DTS_{\lambda}(v)$ exists if and only if $\lambda kv \equiv 0 \pmod{3}$, $v \equiv 1 \pmod{k}$ and $v \geq 3$.

Steiner triple systems, denoted $STS$, have been extensively explored in connection with these types of questions. In particular, a reverse $STS(v)$ admits an automorphism consisting of a fixed point and $(v - 1)/2$ transpositions. A reverse $STS(v)$ exists if and only if $v \equiv 1, 3, 9 \text{ or } 19 \pmod{24}$ [5, 7, 9, 10]. With this result as motivation, we define a reverse $DTS_{\lambda}(v)$ to be one admitting an automorphism consisting of a fixed point and $(v - 1)/2$ transpositions if $v$ is odd, or $v/2$ transpositions if $v$ is even. The purpose of this paper is to use the above mentioned results along with some new constructions to give necessary and sufficient conditions for the existence of a reverse $DTS_{\lambda}(v)$ for all $\lambda \geq 1$. We will take advantage of the fact that if there exists a $DTS_{\lambda_1}(v)$ and a $DTS_{\lambda_2}(v)$ both of which admit $\pi$ as an automorphism, then there exists a $DTS_{\lambda_1+\lambda_2}(v)$ admitting $\pi$ as an automorphism.

2 Reverse Directed Triple Systems With $\lambda = 1$

In this section and the next section we will deal with reverse $DTS_{\lambda}(v)$ on the set $X = \{a, b\} \times \mathbb{Z}/2$ admitting the automorphism $\pi = (a_0, b_0)(a_1, b_1) \cdots (a_{v/2-1}, b_{v/2-1})$. We represent the ordered pair $(x, y)$ as $x_y$.  

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Lemma 2.1. If a reverse $DTS_\lambda(v)$ exists where $v$ is even, then $\lambda v(v-4) \equiv 0 \pmod{24}$.

Proof: Each block of such a $DTS_\lambda(v)$ must be of one of the following forms:

1. $[a_i, a_j, a_k]$ or $[b_i, b_j, b_k]$ where $i, j, k$ are distinct,
2. $[a_i, b_j, b_k]$ or $[b_i, a_j, a_k]$ where $j \neq k$,
3. $[a_i, b_j, a_k]$ or $[b_i, a_j, b_k]$ where $i \neq k$, or
4. $[a_i, a_j, b_k]$ or $[b_i, b_j, a_k]$ where $i \neq j$.

Let $r$ be the number of blocks of type 1, $s$ the number of type 2, $t$ the number of type 3, and $u$ the number of type 4. Notice that $r, s, t$ and $u$ are all even. The number of blocks in a $DTS_\lambda(v)$ is $\lambda v(v-1)/3$ so $r + s + t + u = \lambda v(v-1)/3$. In this $DTS_\lambda(v)$ there is a total of $\lambda v(v-2)/2$ pairs of the form $(a_i, a_j)$ where $\alpha \in \{a, b\} \setminus \{i \neq j\}$. Blocks of the first type each contain 3 such pairs, blocks of the second, third and fourth types each contain 1 such pair. So $3r + s + t + u = \lambda v(v-2)/2$. So $r = \lambda v(v-4)/12$ where $r$ is even.

The conditions for the existence of a $DTS_1(v)$ along with Lemma 2.1 imply that the necessary conditions for the existence of a reverse $DTS_1(v)$ are $v \equiv 0, 1, 3, 4, 7, 9 \pmod{12}$. We now show that these necessary conditions are sufficient.

Theorem 2.1. A reverse $DTS_1(v)$ exists if and only if $v \equiv 0, 1, 3, 4, 7, 9 \pmod{12}$.

Proof: For sufficiency, we present five cases.

Case 1. Suppose that $v \equiv 1$ or $3 \pmod{6}$. Then there exists a $(v-1)/2$–rotational $DTS_1(v)$ by Corollary 1.1. This $DTS_1(v)$ is clearly also reverse.

Case 2. Suppose that $v \equiv 4 \pmod{12}$. Then there exists a cyclic $DTS_1(v)$ admitting an automorphism $\alpha$ which consists of a single cycle of length $v$. The automorphism $\alpha^{v/2}$ consists of $v/2$ transpositions and therefore this $DTS_1(v)$ is also reverse.

Case 3a. Suppose that $v = 24$. Let $\alpha$ be the permutation $(a_0, a_1, \ldots, a_9, b_0, b_1, \ldots, b_9) (a_{10}, a_{11}, b_{10}, b_{11})$. Consider the blocks:

$[\alpha^j(a_{10}), \alpha^j(a_{11}), \alpha^j(b_{11})]$ for $j = 0, 1, 2, 3$, and

$[\alpha^j(a_{10}), \alpha^j(a_1), \alpha^j(a_0)], [\alpha^j(a_2), \alpha^j(a_0), \alpha^j(a_{10})], [\alpha^j(a_1), \alpha^j(a_{10}), \alpha^j(b_8)],$ 
$[\alpha^j(a_3), \alpha^j(a_{10}), \alpha^j(b_9)], [\alpha^j(a_0), \alpha^j(a_1), \alpha^j(a_8)], [\alpha^j(a_0), \alpha^j(a_2), \alpha^j(b_5)],$
$[\alpha^j(a_0), \alpha^j(a_3), \alpha^j(b_2)], [\alpha^j(a_0), \alpha^j(a_4), \alpha^j(b_4)], [\alpha^j(a_0), \alpha^j(a_8), \alpha^j(b_1)]$

for $j = 0, 1, \ldots, 19$. 

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These blocks form a collection of base blocks for a reverse $DTS_1(24)$ under $\pi$.

Case 3b. Suppose that $v \equiv 0 \pmod{24}$, $v \neq 24$. Let $v = 24t$, $t \geq 2$, and let $\alpha$ be the permutation $(a_0, a_1, \ldots, a_{12t-3}, b_0, b_1, \ldots, b_{12t-3})(a_{12t-2}, a_{12t-1}, b_{12t-2}, b_{12t-1})$. Consider the blocks:

\[ [\alpha^j(a_{12t-2}), \alpha^j(a_{12t-1}), \alpha^j(b_{12t-1})] \text{ for } j = 0, 1, 2, 3, \]
\[ [\alpha^j(a_{12t-2}), \alpha^j(a_1), \alpha^j(a_0)] \text{ and } [\alpha^j(a_2), \alpha^j(a_0), \alpha^j(a_{12t-2})] \]
for $j = 0, 1, \ldots, 24t - 5$,
\[ [\alpha^j(a_1), \alpha^j(a_{12t-2}), \alpha^j(b_{12t-4})] \text{ for } j = 0, 1, \ldots, 24t - 5, \]
\[ [\alpha^j(a_3), \alpha^j(a_{12t-2}), \alpha^j(b_{12t-3})] \text{ for } j = 0, 1, \ldots, 24t - 5, \]
\[ [\alpha^j(a_0), \alpha^j(a_1), \alpha^j(a_{10t-2})] \text{ and } [\alpha^j(a_0), \alpha^j(a_{8t-3}), \alpha^j(b_{8t-5})] \]
for $j = 0, 1, \ldots, 24t - 5$,
\[ [\alpha^j(a_0), \alpha^j(a_{4t-3}), \alpha^j(b_{4t-4})] \text{ for } j = 0, 1, \ldots, 24t - 5, \]
\[ [\alpha^j(a_0), \alpha^j(a_{8t-4-2t}), \alpha^j(b_{12t-7-i})] \text{ for } i = 0, 1, \ldots, 4t - 3 \]
and $j = 0, 1, \ldots, 24t - 5$,
\[ [\alpha^j(a_0), \alpha^j(a_{8t-5-2t}), \alpha^j(b_{4t-5-i})] \text{ for } i = 0, 1, \ldots, 2t - 2 \]
and $j = 0, 1, \ldots, 24t - 5$,
\[ [\alpha^j(a_0), \alpha^j(a_{4t-5-2t}), \alpha^j(b_{2t-4-i})] \text{ for } i = 0, 1, \ldots, 2t - 4 \]
and $j = 0, 1, \ldots, 24t - 5$.

These blocks form a collection of base blocks for a reverse $DTS_1(v)$ under $\pi$.

Case 4. Suppose that $v \equiv 12 \pmod{48}$. Let $v = 48t + 12$. Consider the blocks:

\[ [a_i, a_{8t+2+i}, a_{16t+4+i}] \text{ and } [a_{16t+4+i}, a_{8t+2+i}, a_i] \text{ for } i = 0, 1, \ldots, 8t + 1, \]
\[ [a_i, a_{10t+2+i}, a_{14t+2+i}] \text{ for } i = 0, 1, \ldots, 24t + 5 \text{ (omit if } t = 0), \]
\[ [a_i, a_{6t-2j+i}, a_{6t+2+2j+i}] \text{ for } i = 0, 1, \ldots, 24t + 5 \text{ and } j = 0, 1, \ldots, t - 1 \]
(omit if $t = 0$),
\[ [a_i, a_{10t-2j+i}, a_{10t+4+2j+i}] \text{ for } i = 0, 1, \ldots, 24t + 5 \text{ and } j = 0, 1, \ldots, t - 2 \]
(omit if $t = 0$),
\[ [a_i, a_{14t+4+2j+i}, a_{14t-2j+i}] \text{ for } i = 0, 1, \ldots, 24t + 5 \text{ and } j = 0, 1, \ldots, t - 1 \]
(omit if $t = 0$),
\[ [a_i, a_{18t+6+2j+i}, a_{18t+4-2j+i}] \text{ for } i = 0, 1, \ldots, 24t+5 \text{ and } j = 0, 1, \ldots, t-1 \]
(omit if $t = 0$),
\[ [a_i, b_{6t+i-1-j+i}, b_{6t+2+j+i}] \text{ for } i = 0, 1, \ldots, 24t + 5 \text{ and } j = 0, 1, \ldots, 6t + 1, \]
\[ [a_i, b_{18t+i+6-j+i}, b_{18t+i+4-j+i}] \text{ for } i = 0, 1, \ldots, 24t + 5 \text{ and } j = 0, 1, \ldots, 6t. \]

These blocks form a collection of base blocks for a reverse \( DTS_1(v) \) under \( \pi \).

Case 5. Suppose that \( v \equiv 36 \pmod{48} \). Let \( v = 48t + 36 \). Consider the blocks:

\[ [a_i, a_{6t+6+i}, a_{16t+12+i}] \text{ and } [a_i, a_{6t+12+i}, a_{6t+6+i}, a_i] \text{ for } i = 0, 1, \ldots, 8t + 5, \]
\[ [a_i, a_{6t+5+i}, a_{10t+8+i}] \text{ for } i = 0, 1, \ldots, 24t + 17, \]
\[ [a_i, a_{6t+3-j+i}, a_{6t+6+j+i}] \text{ for } i = 0, 1, \ldots, 24t + 17 \text{ and } j = 0, 1, \ldots, 2t - 1 \]
(omit if \( t = 0 \)),
\[ [a_i, a_{10t+7-j+i}, a_{10t+9+j+i}] \text{ for } i = 0, 1, \ldots, 24t + 17 \text{ and } j = 0, 1, \ldots, 2t, \]
\[ [a_i, b_{12t+8+i}, b_{18t+i+12+i}] \text{ for } i = 0, 1, \ldots, 24t + 17, \]
\[ [a_i, b_{22t+15+i}, b_{22t+16+i}] \text{ for } i = 0, 1, \ldots, 24t + 17, \]
\[ [a_i, b_{6t+14-j+i}, b_{6t+3-j+i}] \text{ for } i = 0, 1, \ldots, 24t + 17 \text{ and } j = 0, 1, \ldots, 6t + 3, \]
\[ [a_i, b_{18t+i+13+j+i}, b_{18t+i+11-j+i}] \text{ for } i = 0, 1, \ldots, 24t + 17 \text{ and } j = 0, 1, \ldots, 4t + 1, \]
\[ [a_i, b_{22t+17+j+i}, b_{14t+9-j+i}] \text{ for } i = 0, 1, \ldots, 24t + 17 \text{ and } j = 0, 1, \ldots, 2t. \]

These blocks form a collection of base blocks for a reverse \( DTS_1(v) \) under \( \pi \).

3 Reverse Directed Triple Systems With \( \lambda > 1 \)

Finally, we give necessary and sufficient conditions for the existence of a reverse \( DTS_\lambda(v) \) where \( \lambda > 1 \).

**Theorem 3.1.** A reverse \( DTS_\lambda(v) \), where \( v \) is odd, exists if and only if \( \lambda v(v-1) \equiv 0 \pmod{3} \). A reverse \( DTS_\lambda(v) \), where \( v \) is even, exists if and only if \( \lambda v(v-1) \equiv 0 \pmod{3} \) and \( \lambda v(v-4) \equiv 0 \pmod{24} \), \( v \neq 2 \).

**Proof:** The necessary conditions follow from the conditions for the existence of a \( DTS_\lambda(v) \) along with Lemma 2.1. We show sufficiency in the following cases.

Case 1. Suppose that \( v \equiv 0, 1, 3, 4, 7, \) or \( 9 \pmod{12} \). Then there exists a reverse \( DTS_1(v) \) by Theorem 2.1. Therefore there exists a reverse \( DTS_\lambda(v) \) for all \( \lambda \geq 1 \).
Case 2. Suppose that $v \equiv 2 \pmod{12}$. Then it is necessary that $\lambda \equiv 0 \pmod{6}$. In this case, there is a $DTS_{\lambda}(v)$ admitting a cyclic automorphism $\alpha$. The automorphism $\alpha^{v/2}$ consists of $v/2$ transpositions and therefore this $DTS_{\lambda}(v)$ is also reverse.

Case 3. Suppose that $v \equiv 5 \pmod{6}$. Then there exists a $(v-1)/2$-rotational $DTS_{\lambda}(v)$ by Corollary 1.1. This $DTS_{\lambda}(v)$ is clearly reverse.

Case 4a. Suppose that $v = 6$. Consider the blocks:

\[
[a_0, b_1, a_2], \ [a_1, b_0, a_2], \ [a_2, b_0, a_1], \ [a_1, a_0, b_0], \ [a_2, a_1, b_1], \\
[a_2, a_0, b_2], \ [b_1, a_0, a_1], \ [b_2, a_1, a_2], \ [b_0, a_2, a_0], \ [a_0, a_1, a_2].
\]

These blocks form a collection of base blocks for a reverse $DTS_2(6)$. Therefore there exists a reverse $DTS_{\lambda}(6)$ for all $\lambda \equiv 0 \pmod{2}$.

Case 4b. Suppose that $v \equiv 6 \pmod{24}$, $v \neq 6$, say $v = 24t + 6$, $t \geq 1$. Consider the blocks:

\[
[a_i, a_{6t+j+i}, a_{6t+1+j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, t - 1, \\
[a_i, a_{5t+j+i}, a_{7t+3+j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, t - 2 \\
\text{(omit if } t = 1), \\
[a_i, a_{2+2j+i}, a_{10t+3+j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, 2t - 2, \\
[a_i, a_{7t+2+i}, a_{7t+1+i}] \text{ for } i = 0, 1, \ldots, 12t + 2, \\
[a_i, a_{4t+1+i}, a_{8t+2+i}] \text{ and } [a_i, a_{8t+2+i}, a_{4t+1+i}] \text{ for } i = 0, 1, \ldots, 8t + 1, \\
[a_i, b_{10t+3+j+i}, a_{8t+4+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, 4t - 1, \\
[a_i, b_{2t+1+j+i}, a_{4t+2+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, 2t - 1, \\
[a_i, b_{4t+4+j+i}, a_{8t+3+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, 2t - 1, \\
[a_i, b_{6t+2+j+i}, a_{2+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, 2t - 1, \\
[a_i, b_{8t+3+j+i}, a_{4t+3+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 2 \text{ and } j = 0, 1, \ldots, 2t - 1, \\
[a_i, b_{10t+3+i}, b_{2+i}] \text{ for } i = 0, 1, \ldots, 12t + 2, \\
[a_i, b_{4t+1+i}, b_{6t+2+i}] \text{ for } i = 0, 1, \ldots, 12t + 2, \text{ and} \\
[a_i, b_{8t+2+i}, b_{6t+1+i}] \text{ for } i = 0, 1, \ldots, 12t + 2.
\]

These blocks form a collection of base blocks for a reverse $DTS_2(v)$. Therefore there exists a reverse $DTS_{\lambda}(v)$ for all $\lambda \equiv 0 \pmod{2}$.

Case 5. Suppose that $v \equiv 8 \pmod{12}$. Then it is necessary that $\lambda \equiv 0 \pmod{3}$. Under these conditions, there is a cyclic $DTS_{\lambda}(v)$ and this $DTS_{\lambda}(v)$ is also reverse by the argument of Case 2.
Case 6. Suppose that \( v \equiv 10 \pmod{12} \). Then it is necessary that \( \lambda \equiv 0 \pmod{2} \). Under these conditions, there is a cyclic \( DTS_\lambda(v) \) and this \( DTS_\lambda(v) \) is also reverse by the argument of Case 2.

Case 7a. Suppose that \( v = 18 \). Consider the blocks:

\[
[a_i, a_{5+i}, a_{6+i}] \text{ and } [a_i, a_{6+i}, a_{3+i}] \text{ for } i = 0, 1, 2, 3, 4, 5, \text{ along with }
\]

\[
[a_i, a_{7+i}, a_{8+i}], [a_i, b_i, b_{1+i}], [a_i, b_{1+i}, b_{3+i}], [a_i, b_{2+i}, b_{6+i}], [a_i, b_{3+i}, b_{8+i}],
[a_i, b_{5+i}, b_{4+i}], [a_i, b_{8+i}, b_{6+i}], [a_i, b_{4+i}, b_{4+i}], [a_i, b_{5+i}, b_{7+i}], \text{ and }
[a_i, b_{2+i}, b_{7+i}] \text{ for } i = 0, 1, \ldots, 8.
\]

These blocks form a collection of base blocks for a reverse \( DTS_2(18) \). Therefore there exists a reverse \( DTS_\lambda(18) \) for all \( \lambda \equiv 0 \pmod{2} \).

Case 7b. Suppose that \( v \equiv 18 \pmod{24} \), \( v \neq 18 \), say \( v = 24t + 18 \), \( t \geq 1 \). Consider the blocks:

\[
[a_i, a_{6t+3+j+i}, a_{6t+5+j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, t - 1,
\]

\[
[a_i, a_{6t+3+j+i}, a_{7t+7+j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, t - 2
\text{ (omit if } t = 1),
\]

\[
[a_i, a_{10t+6-j+i}, a_{10t+9+j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, 2t - 1,
\]

\[
[a_i, a_{7t+5+i}, a_{7t+6+i}] \text{ for } i = 0, 1, \ldots, 12t + 8,
\]

\[
[a_i, a_{6t+4+i}, a_{10t+8+i}] \text{ for } i = 0, 1, \ldots, 12t + 8,
\]

\[
[a_i, a_{4t+3+i}, a_{8t+6+i}] \text{ and } [a_i, a_{8t+6+i}, a_{4t+3+i}] \text{ for } i = 0, 1, \ldots, 8t + 5,
\]

\[
[a_i, b_{10t+8+j+i}, a_{8t+8+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, 4t + 1,
\]

\[
[a_i, b_{2t+2+j+i}, a_{4t+4+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, 2t,
\]

\[
[a_i, b_{4t+3+j+i}, a_{8t+7+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, 2t,
\]

\[
[a_i, b_{8t+5+j+i}, a_{2t+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, 2t,
\]

\[
[a_i, b_{6t+7+j+i}, a_{4t+5+2j+i}] \text{ for } i = 0, 1, \ldots, 12t + 8 \text{ and } j = 0, 1, \ldots, 2t,
\]

\[
[a_i, b_{10t+8+i}, b_{2t+1+i}] \text{ for } i = 0, 1, \ldots, 12t + 8,
\]

\[
[a_i, b_{8t+1+i}, b_{6t+4+i}] \text{ for } i = 0, 1, \ldots, 12t + 8, \text{ and }
\]

\[
[a_i, b_{4t+3+i}, b_{6t+5+i}] \text{ for } i = 0, 1, \ldots, 12t + 8.
\]

These blocks form a collection of base blocks for a reverse \( DTS_2(v) \). Therefore there exists a reverse \( DTS_\lambda(v) \) for all \( \lambda \equiv 0 \pmod{2} \). \( \square \)

Theorem 3.1 gives a complete classification of reverse directed triple systems.

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References


