but never gets there." Expressed in the appropriate "mathemate" we have the following:

**Faux Definition.** Let \( f(x) \) be a function defined for all \( x \in (-\infty, \infty) \). Then, the line \( y = k \) is a horizontal asymptote of \( f \) if, as \( x \) increases, the graph of \( f \) gets closer and closer to \( y = k \) but \( f(x) \neq k \) for all \( x \in (-\infty, \infty) \).

We can easily represent several flaws in this "definition." We do so in the following two examples.

**Example 1.** Suppose that a weight is attached to a vertically mounted spring and that the weight is set in motion by displacing it vertically from its equilibrium position and then releasing it. If we let \( f(t) \) be the displacement of the weight from its equilibrium position at time \( t \), then, assuming the presence of friction and with the appropriate choice of constants and units, we find that \( f(t) = e^{-\frac{t}{8}} \cos 4t \). (See fig. 1.) Notice that \( f \) has a horizontal asymptote of \( y = 0 \). Also, \( f \) takes on the value 0 for all values of \( t \) such that

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t = \frac{2n + 1}{\pi}
\]

for some integer \( n \). Certainly this example illustrates that \( f \) can "get there!" This example also illustrates the falseness of the "gets closer and closer" assumption.

Example 1, which illustrates the so-called damped harmonic motion, is especially appealing because of its physical origin. Students can easily visualize masses oscillating on springs. A less physically interesting, but equally valid, example is the following.

**Example 2.** Consider \( g(x) = e^{-x} \) and the line \( y = -1 \). (See fig. 2.) Since \( g \) is a decreasing positive function, it does in fact get closer and closer to the line \( y = -1 \) as \( x \) increases. Also, \( g \) never attains the value \( -1 \). However, \( y = -1 \) is not a horizontal asymptote of \( g \).

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**Horizontal asymptotes: What they are not**

A common misconception about a horizontal asymptote of a function is that the function "gets closer and closer to the asymptote" we have the following:

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