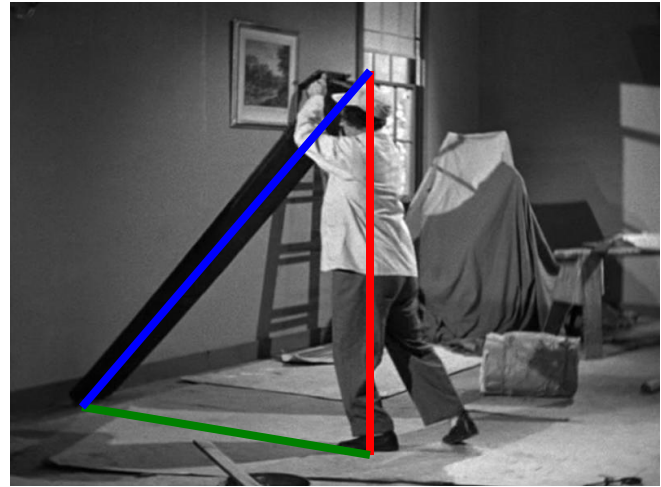


# The Three Stooges and Trigonometry

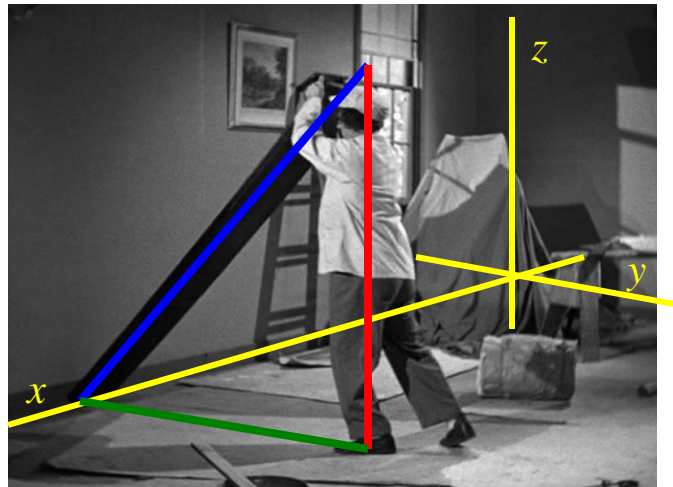


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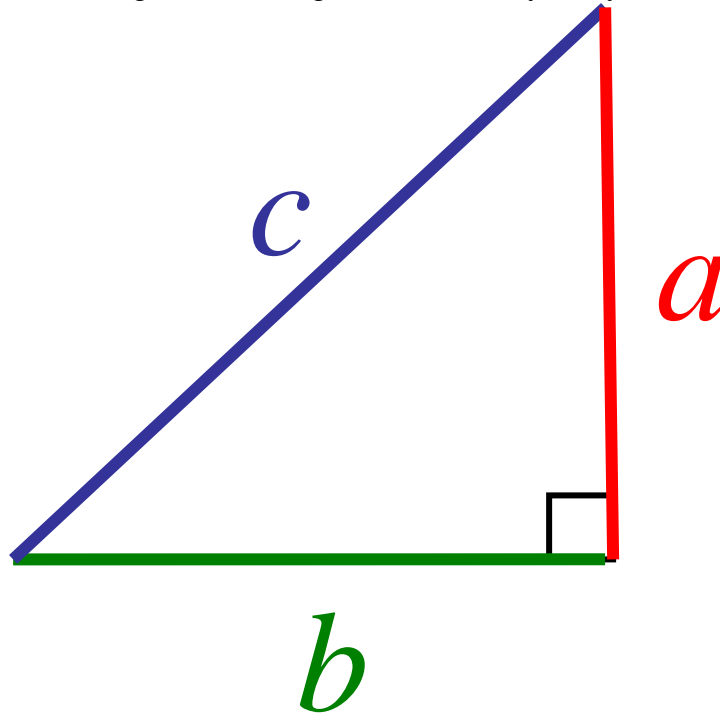
In the Three Stooges film *A Bird in the Head*, middle Stooge Larry props a board up against a wall, as seen here:



First, we want to find the length of the board. We draw a triangle with the board as one side, Larry as another side, and the final side is from the bottom of the board to Larry's foot. Since Larry is standing vertically, then this is a right triangle. However, we are viewing it in perspective, so it does not appear to be a right triangle. Next, we use the walls of the room to introduce a 3-dimensional coordinate system:



We now use this coordinate system and projective geometry (or appropriate computer software) to rotate the triangle around so that it appears in a plane perpendicular to our line of site and hence appears as a right triangle. The rotation is about the edge of the triangle determined by Larry. This results in the triangle:



Measure the lengths of each side of the triangle in centimeters (to the nearest 0.1 cm) and record the results here (don't forget to include the units of centimeters in your answer):

Side	Length
<i>a</i>	7.3 cm
<i>b</i>	8.1 cm
<i>c</i>	10.8 cm

According to *Stooges Among Us*, edited by Lon & Debra Davis, BearManor Media (2008): "... like Moe, Larry was only five-feet-four-inches tall" (page 10). This measurement can be used to scale the right triangle above. First, we need to decide on a single unit with which to measure distances in the scene. Let's use inches. We know that 12 inches equals 1 foot, so this allows us to introduce the *conversion factor* (12 inches)/(1 foot) = (12"/1'). Use this conversion factor to convert Larry's height (5'4") to inches:

$$5'4'' = 5' + 4'' = 5' \times (\text{conversion factor}) + 4''$$

$$= \underline{5' \times 12''/1' + 4' = 64''} .$$

The *scale factor* of the photograph can be determined based on the value of the length of side *a*, which you entered in the table above and the fact that this length corresponds to the height of Larry which you have just calculated in inches. The scale factor is:

$$\begin{aligned} & (\text{Height of Larry in inches})/(\text{length of side } a \text{ in centimeters}) \\ &= \frac{64 \text{ inches}}{7.3 \text{ cm}} \\ &= \underline{8.77 \text{ inches/cm.}} \end{aligned}$$

The scale factor must be distinguished from a conversion factor! The scale factor is particular to this exercise. In general, lengths of objects can be measured in inches or centimeters and we can *convert* from inches to centimeters or back. Such a conversion requires a *conversion factor* (such as used above to convert feet to inches). The relevant conversion factors in this case are 2.54 cm/inch and 0.3937 inch/cm. However, these values are irrelevant to this exercise and are unrelated to the scale factor.

Now use the scale factor from above to convert the lengths of sides *a*, *b*, and *c* (from the first table) into inches. The scaled length of side *a* should be Larry's height in inches. Don't forget to include units:

Side	Length	Scaled Length
<i>a</i>	7.3 cm	64.0"
<i>b</i>	8.1 cm	71.0"
<i>c</i>	10.8 cm	94.7"

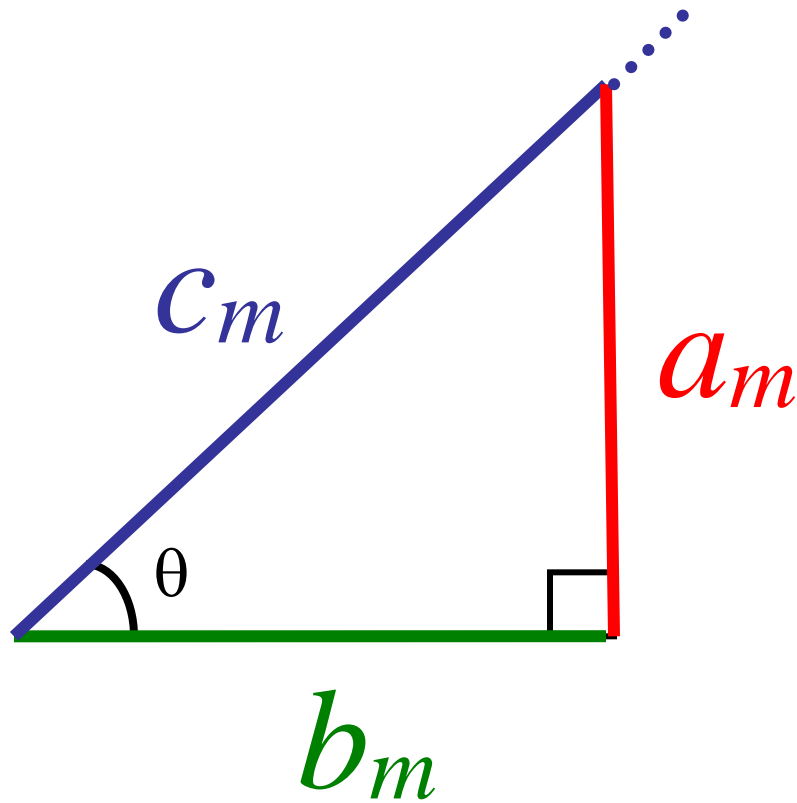
You can now tell what the length of the board is:

$$\text{Length of the board} = \underline{94.7''}.$$

When the board falls and hits Moe on the head, assume that the point on the board which hits Moe's head is 6 inches from the end of the board.



This leads to a new triangle which reflects the situation right when the board hits Moe in the head. We denote the sides of this triangle with subscripts of "m" (for "Moe"):



You know that side  $a_m$  is the height of Moe (who is the same height as Larry), and you know the length of the board from above. Side  $c_m$  is 6 inches less than the length of the board. Use this information and the Pythagorean Theorem to record the lengths in the following table (include units):

Side	Scaled Length
$a_m$	64.0''
$b_m$	61.4''
$c_m$	88.7''

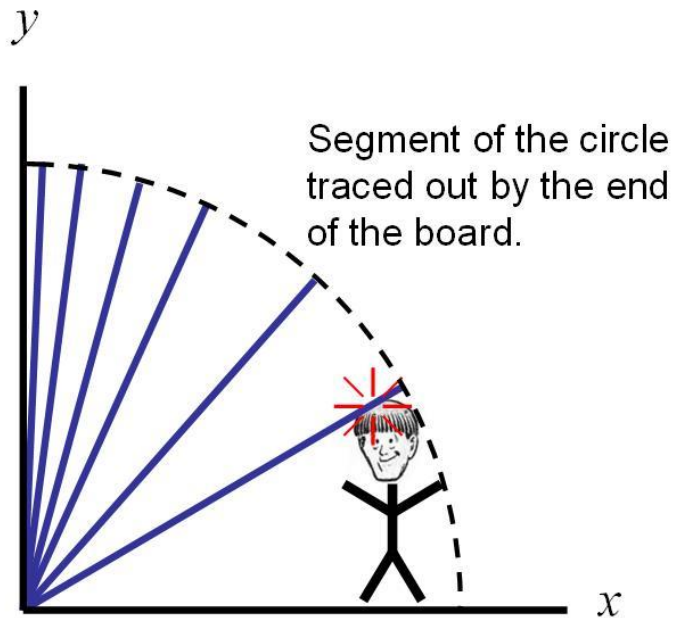
We now turn our interest to the angle  $\theta$  between the board and the floor at the time when the board hits Moe in the head. Since you have a right triangle containing  $\theta$ , you can find each of the six trigonometric functions of  $\theta$ . Give each to four decimal places.

Trig Function	Value
$\cos \theta$	$66.4''/88.7'' = 0.6922$
$\sin \theta$	$64.0''/88.7'' = 0.7215$
$\tan \theta$	$64.0''/61.4'' = 1.0423$
$\sec \theta$	$88.7''/61.4'' = 1.4446$
$\csc \theta$	$88.7''/64.0'' = 1.3859$
$\cot \theta$	$61.4''/64.0'' = 0.9594$

Use an inverse trigonometric function to find angle  $\theta$  (give your answer in degrees, to the nearest 0.1 degree):

$$\theta = \underline{\tan^{-1}(1.0423) = 46.2^\circ}.$$

Now we introduce  $x$ - $y$  coordinates with the origin at the point where the board meets the wall:



Use this coordinate system to answer the following questions. Express coordinates in units of inches.

1. What are the coordinates of the top end of the board when it is standing vertically?

$$(x, y) = \underline{(0'', 94.7'')}.$$

2. What are the coordinates of the top of Moe's head?

$$(x, y) = \underline{(b_m, a_m) = (61.4'', 64.0'')}.$$

3. What are the coordinates of the end of the board when the board hits Moe on the head?

$$(x, y) = \underline{(94.7'' \cos(46.2^\circ), 94.7'' \sin(46.2^\circ)) = (65.5'', 68.4'')}.$$

4. What is the equation of the circle which is traced out by the end of the board?

$$\underline{x^2 + y^2 = (94.7'')^2 .}$$

5. Convert  $\theta$  to radians. Use the conversion factor  $\pi$  radians /180°.

$$\theta = \underline{(46.2^\circ)(\pi/180^\circ) = 0.8063 .}$$

6. Through what angle did the board travel, from when it was vertical to when it hit Moe? Give your answer in radians.

$$\text{angle} = \underline{\alpha = \pi/2 - \theta = 0.7645 .}$$

7. How far did the end of the board travel from when it was vertical to when it hit Moe?

$$\text{distance} = \underline{r\alpha = (94.7'')(0.7645) = 72.40'' .}$$

8. What area did the board sweep out from when it was vertical to when it hit Moe?

$$\text{area} = \underline{r^2\alpha/2 = (94.7'')^2(0.7645)/2 = 3428.05 \text{ in}^2 .}$$