Archimedes—2000 Years Ahead of His Time

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(an updated version of a Spring, 2011 presentation)

[SLIDE: Codex Book] The primary reference for this presentation is *The Archimedes Codex—How a Medieval Prayer Book is Revealing the True Genius of Antiquities Genius of Antiquities Greatest Scientist* by Reviel Netz and William Noel, 2007.

[SLIDE: Infinite Secrets] Some of the story is contained in the NOVA episode *Infinite Secrets: The Genius of Archimedes* from 2003.

[SLIDE: Works, Heath] The classical English presentation of Archimedes work is in *The Works of Archimedes*, edited by Sir Thomas Heath, first published in 1897.

[SLIDE] Archimedes—A Biography

Archimedes lived from about 287 BCE to 212 BCE. He is, perhaps, most widely known for:

[SLIDE: Eureka] (1) Jumping out of his bathtub, shouting "*Eureka*," and running through the streets naked,

[SLIDE: Lever] (2) For saying that he could move the Earth itself given a lever, fulcrum, and place to stand, and

[SLIDE: Disney] (3) Co-starring in Disney's 1963 *The Sword in the Stone.*

[SLIDE: Vitruvius] Did he *really* cry "Eureka"? The most famous version of this story (also the earliest) was told by Vitruvius. However, Vitruvius writes some 175 years after Archimedes' death. This story really deals with an application of Archimedes' idea of buoyancy to detect a forgery in a crown made for Hiero of Syracuse. Archimedes is alleged to have noticed that when he entered a full tub of water, the amount of water displaced as he entered the tub was equal to the volume of Archimedes which was in the tub. This allowed him to test the crown to see if it was indeed made of gold, or if some cheaper metal had been used for part of the crown. It turned out that the goldsmith in fact had used a cheaper metal and the crown was not made of pure gold. The scientific ideas here are from Archimedes' work *Floating Bodies.* However, it is very likely that the story of Archimedes running naked through the streets is more of an urban legend, than accurate history.

[SLIDE: Weapons] Archimedes developed a number of practical devices, included weapons of war. He proposed construction of giant claws on the walls of Syracuse that were used against the Romans in the siege of Syracuse. Plutarch reports that the Romans were so terrorized by what they had heard about Archimedes that "if they did but see a piece of rope or wood projecting above the wall, they would cry 'there it is', declaring that Archimedes was setting some engine in motion against them, and would turn their backs and run away." Archimedes also made improvements on the catapult by making it a variable range device.

[SLIDE: Screw] On a less militaristic side, Archimedes is credited with the invention of the so-called Archimedes screw which is still in use in parts of the world to help irrigate crops.

[SLIDE: Area/Volume] In geometry Archimedes's work consisted mainly of studies of the area (or quadrature) and volume (or "cubature") of curved planer figures and solids with curved surfaces. This work begins where Euclid's Book XII ends. In this sense, Archimedes is really the next great geometer in history after Euclid. His work in mechanics and mechanical inventions has a relationship to his work in geometry, as revealed in his treatise *The Method*.

[SLIDE: Extant Works] The surviving works of Archimedes (in the order of appearance as estimated by Sir Thomas Heath) are:

- *On Plane Equilibriums*, Book I.
- *Quadrature of the Parabola*.
- *On Plane Equilibriums*, Book II.
- *The Method*.
- *On the Sphere and Cylinder*: Two Books.
- *On Spirals*.
- *On Conoids and Spheroids*.

- *On Floating Bodies*: Two Books.
- *Measurement of a Circle*.
- *The Sand-Reckoner* (Psammites).
- *Stomachion* (a fragment).

In addition, there are several other works known from secondary sources which reference Archimedes.

[SLIDE: Method] It is *The Method* which most interests us today. Most of the math that survives from the Greek world is written in a very dry way. It starts with statements of definitions, and sometimes axioms and postulates, and then dryly states and proves results. In fact, Euclid's *Elements of Geometry* proceeds in this way (as does Apollonius' *Conic Sections*). Archimedes' *Method* is almost unique in that it describes the thought process Archimedes used to arrive at some of his results.

[SLIDE: Death] Archimedes was killed during the Second Punic War while the Romans captured Syracuse. The legend of Archimedes' death is famous and reported by Plutarch.

[SLIDE: Plutarch] "Archimedes refused to go until he had worked out his problem and established its demonstration, whereupon the soldier flew into a passion, drew his sword, and killed him." Archimedes was 75 years old at the time.

[Slide: Library/Hypatia] After the decline of the Greek sciences (the end defined by the burning of the great library in Alexandria and the murder of Hypatia in 415 CE), interest in mathematics in Europe basically died. However, many of Archimedes works survived the centuries, copied by scribes.

[Slide: The Archimedes Palimpsest]

[SLIDE: Scribe] In the tenth century (likely in the third quarter of the tenth century), a scribe copied some of Archimedes' work, including *On Plane Equilibriums*, *On the Sphere and Cylinder*, *Measurement of a Circle*, *On Spiral*, *On Floating Bodies*, *The Method* and *Stomachion*. The writing medium was sheep skin (i.e., parchment).

[SLIDE: Archimedes Recycled] In the 12th century, a monk at the Marsaba Monestary in the Judean desert ran out of parchment while copying a book of prayers! **[INSERT: Palimpsest Cosntruction]** He reached for the copy of Archimedes work. He removed the pages of the work, scrapped them clean, rotated them 90°, wrote the book of prayers over the work of Archimedes, and folded the pages to create the prayer book.

[SLIDE: "Palimpsest"] This is not a unique incident, and such a book is called a *palimpsest*. Derived from the Greek words *palin* (again) and *psan* (to rub).

[SLIDE: Heiberg] The location of the palimpsest was not known, until it turned up in a library in Constantinople. The classicist Johan Ludwig Heiberg heard of the book through the library catalog, and in 1906 went to Constantinople. He had detailed photographs of the book taken and he worked from these. The only instrument he used to explore the originals was a magnifying glass.

[SLIDE: Heath] Sir Thomas Heath is responsible for translating much of Heiberg's work into English. He is the one, for example, who published the definitive English copy of Euclid's *Elements of Geometry.* In 1897 he translated Heiberg's work on Archimedes.

[SLIDE: Question] After Heiberg's visit to Constantinople, the palimpsest seems to have disappeared (possibly lost in the chaos of World War 1). This was the last to be heard of the palimpsest for several decades.

[SLIDE: Wilson] In 1971, Nigel Wilson heard about a single page of a manuscript in a library in Cambridge, England.

[SLIDE: Titchendorf] It seems that at some point while traveling in Constantinople, Constantine Titchendorf tore a page from the palimpsest and brought it back to England. He was a biblical expert, but noticed that the work might be important. Of course, this violates the trust of the library and is highly unprofessional. Nigel Wilson inspected the page with an ultraviolet light which revealed much more detail than possible with Heiberg's magnifying glass.

[SLIDE: Oyens] In 1991, Felix de Marez Oyens of Christies Auction House (London) got a letter from a family in Paris. The family claimed that one of their members was traveling in Turkey in the 1920s and "acquired" a book in Constantinople. It had been in a Paris apartment since then. This book was the lost Archimedes Palimpsest!

[**SLIDE: Condition/Video**] The palimpsest was in horrible condition! There were places where it had molded through, there was glue on parts of it, and it had candle wax on it from centuries of reading. This is not to mention the fact that the work of Archimedes was underneath the easily-visible writing of the pray book! In addition, parts of Archimedes was obscured by the binding itself. None-the-less, it sold at auction for $2,200,000 in 1998 to an anonymous bidder, today only known as "Mr. B." Some have speculated that it might be Warren Buffet or Bill Gates.

[**SLIDE: Noel**] William Noel of the Walters Art Museum in Baltimore, contacted Christies with a request to borrow the palimpsest from the owner. He was granted full access to the book. He put together a team of researchers who have been working on translating and preserving the book since 1999.

[**SLIDE: Infinite Secrets**] The story of this project is given in the 2003 NOVA episode *Infinite Secrets: The Genius of Archimedes*. This video gives some history about Archimedes himself and includes interviews of several members of the Walters Art Museum research team. You can watch this video online through YouTube. The Public Broadcasting System also has a resource website for this documentary.

[**SLIDE: Website**] Work at the Walters Art Museum can be followed at the Archimedes Palimpsest Project online at this web address. There are several videos, history, and an ever-growing digital version of the results of image enhancement of the pages of the palimpsest.

[SLIDE: Two Volumes] In late 2011, Cambridge University Press released a two volume study of the Archimedes palimpsest. These include full color images and descriptions of the technical way in which the images were made. Complete transcripts of the included texts are included. **[INSERT QUOTE]** A *Washington Post* review of these works states: "This is the iceberg in full view, a massive tome that took more than a decade to produce, recovering—perhaps as fully as can ever be hoped—texts that miraculously escaped the oblivion of decay and destruction."

[Slide: Archimedes and π]

[SLIDE: Euclid] Let's start with the definition of π. Euclid's *Elements*, Book XII, Proposition 2 states that: *Circles are to one another as the squares on the diameters.* That is, the area of a circle is proportional to the square of its diameter, or equivalently, "the area of a circle is proportional to the square of its radius." We call this constant of proportionality π.

[SLIDE: Proposition 1] Archimedes' *Measurement of a Circle* starts without fanfare and begins with a proof of the familiar formula for the area of a circle: **Proposition 1.** "The area of any circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius, and the other to the circumference of the circle." That is, a circle of radius r, and hence circumference $2\pi r$, has area πr^2. This result relates the same constant of proportionality,

π, to both the circumference and area of a circle.

SLIDE: Triangle] Let K be the triangle described. If the area of the circle is not equal to the area of K, then it must be either greater or less in area. Archimedes will now apply the method of exhaustion to show that the area of the circle can neither be greater than nor less than the area of K. You will notice the parallel between this technique of proof and an ε proof. We present his argument in two parts.

[**SLIDE: Part I**] For the first part: If possible, let the area of the circle be greater than the area of triangle K. Since the area of a planar object can easily be computed if the object has straight line segments as its edges, Archimedes will restrict himself to these types of objects when calculating precise areas. That is, he will base his area computations on squares and triangles.

[**SLIDE: Square**] Inscribe a square in the circle using points A, B, C, and D.

[**SLIDE: Octagon**] Now [**INSERT: Remove Square**], bisect the arcs AB, BC, CD, and DA [**INSERT: Octagon**]. Use these points [**INSERT: Octagon**] to create a regular octagon inscribed in the circle.

[**SLIDE**] Continue this process [**INSERT: Points**] until the area of the resulting polygon P is greater than the area of triangle K. [**INSERT: Polygon**] Notice the implicit assumption here that increasing the number of sides in the inscribed regular polygon is sufficient to produce a polygon with an area close to the area of the circle. This reflects a common practice in Greek geometry of depending on the use of pictures to draw conclusions about the objects under discussion.

[**SLIDE**]Consider such a general polygon P. Let AE be a side of polygon P. [**INSERT** A **AND** E] Let N be the midpoint of side AE. [**INSERT POINT** N] [**REMOVE WORDS**] Let O be the center of the circle. [**INSERT CENTER**] Introduce line segment ON. [**INSERT SEGMENT** ON]

[**SLIDE**] Archimedes observes that line segment ON [**INSERT COMMENT**] is shorter than the radius of the circle, r (both in green), and that the perimeter of polygon P is less than the circumference of the circle, $2\pi r$ (both in blue). Again, he is depending on the picture for some of the details of his proof.

[**SLIDE**] Now for some computations. If polygon P has n edges, then the area of the polygon equals $2n$ times the area of triangle T. Since T is a right triangle and its area is one-half base times height, then the area of this triangle if one-half NE times ON. Next, we cancel the 2 and the 1/2, and observe that n times NE is the perimeter of the regular polygon P. From our previous observations that the perimeter of P is less than $2\pi r$ and that ON is less than r, we see that the area of the polygon is less than the area of the original triangle K. Here we, like Archimedes, do not distinguish between line segment ON and the *length* of line segment ON. However, Archimedes skips some steps and does not show this level of detail in his proof.

[**SLIDE**] To summarize: [**INSERT ASSUMPTION**] We assumed that the area of the circle was greater than the area of triangle K. **[INSERT INTERMEDIATE STEP]** As an intermediate step, we inscribed a polygon P with area greater than the area of triangle K. **[INSERT CONCLUSION]** We conclude that the inscribed polygon P has area less than triangle K.

[SLIDE] Since the polygon is inscribed, we know that the area of the circle is not greater than the area of triangle K. In other words, the area of the circle is less than or equal to πr^2.

[SLIDE] For the second part: If possible, let the area of the circle be less than the area of triangle K. Archimedes proceeds in a similar way as above.

[SLIDE] To summarize: **[INSERT ASSUMPTION]** We assume that the area of the circle is less than the area of triangle K. **[INSERT INTERMEDIATE STEP]** As an intermediate step, we circumscribe a polygon P with area less than the area of triangle K. **[INSERT CONCLUSION]** We conclude that the inscribed polygon P has area greater than triangle K.

[SLIDE] Since the polygon is circumscribed, we know that the area of the circle is not less than the area of triangle K. In other words, the area of the circle is greater than or equal to πr^2.

[SLIDE] Therefore, Archimedes has established for the first time that the area of a circle is πr^2.

Archimedes and the Approximation of π

[SLIDE: PROPOSITION 3] We have the definition of π as the ratio of the circumference of a circle to the circle's diameter. In Proposition 3 of *Measurement of a Circle*, Archimedes restricts the value of π to between $3\frac{1}{7}$ and $3\frac{10}{71}$.

[SLIDE: $\sqrt{3}$] In the proof of Proposition 3, Archimedes gives upper and lower bounds on $\sqrt{3}$. Without any explanation to his reasoning, he states that $\frac{265}{153} < \sqrt{3} < \frac{1351}{780}$.

[SLIDE: CIRCLE] To get the upper bound on π, Archimedes constructs a 96-sided polygon circumscribed around the circle. He starts with a circle with diameter AB. He constructs a tangent to the circle at point C and a central angle in the circle of 30°.

[SLIDE] Since triangle OCA is a 30-60-90 right triangle, the Pythagorean Theorem allows us to conclude that $\frac{OA}{AC} = \sqrt{3}$ and so is greater than $\frac{265}{153}$. Similarly, $\frac{OC}{AC} = 2 = \frac{306}{153}$.

[SLIDE] Bisect the 30° angle, a standard straight-edge and compass construction. Call the resulting 15° angle AOD, as drawn.

[SLIDE] By Proposition 3 of Euclid's *Elements* Book VI, we have that $\frac{OC}{OA} = \frac{CD}{AD}$.

[SLIDE] Adding 1 to both sides of the previous equality give $\frac{OC + OA}{OA} = \frac{CD + AD}{AD} = \frac{AC}{AD}$.

[SLIDE] Cross multiplying in the first and last expressions of this equation gives $\frac{OC + OA}{AC} = \frac{OA}{AD}$.

[SLIDE] This new equation gives that $\dfrac{OA}{AD} = \dfrac{OC}{AC} + \dfrac{OA}{AC}$ which, based on the properties of the 30-60-90 triangle, equals $\dfrac{\sqrt{3}}{1} + \dfrac{2}{1}$ which, using Archimedes' approximation of $\sqrt{3}$, is greater than $\dfrac{571}{153}$.

[SLIDE] Since triangle AOD is a right triangle, the Pythagorean Theorem gives that $OD^2 = OA^2 + AD^2$, and so $\dfrac{OD^2}{AD^2} = \dfrac{OA^2 + AD^2}{AD^2}$. Given the bound on $\dfrac{OA}{AD}$, we can conclude that $\dfrac{OD^2}{AD^2} > \dfrac{349,450}{23,409}$.

[SLIDE] Since $349,450 > \left(591\frac{1}{8}\right)^2$, an approximation Archimedes uses with no explanation, then we conclude that $\dfrac{OD}{AD} > \dfrac{591\frac{1}{8}}{153}$. We should comment here that at the time of Archimedes, there was no such thing as a decimal representation of numbers. In fact, the so-called Arabic numerals which we use were still 1,000 to 1,400 years in the future. There was no consideration of negative numbers and zero was not considered a number. The existence of irrational numbers *was* recognized, though.

[SLIDE] Next, bisect the 15° angle, creating 7.5° angle AOE, as shown.

[SLIDE] Again, by Euclid's Book VI, Proposition 3, $\dfrac{OD}{OA} = \dfrac{DE}{AE}$.

[SLIDE] Similar to the previous computation, we have $\dfrac{OD + OA}{OA} = \dfrac{DE + AE}{AE} = \dfrac{AD}{AE}$.

[SLIDE] Cross multiplying gives $\dfrac{OD + OA}{AD} = \dfrac{OA}{AE}$.

[SLIDE] We established above that $\dfrac{OD}{AD} > \dfrac{591\frac{1}{8}}{153}$ and $\dfrac{OA}{AD} > \dfrac{571}{153}$. These give is from the previous equation that $\dfrac{OA}{AE} > \dfrac{1162\frac{1}{8}}{153}$.

[SLIDE] Using this inequality with, as above, the Pythagorean Theorem as applied to right triangle AOE gives $\dfrac{OE^2}{AE^2} > \dfrac{1,373,943\frac{33}{64}}{23,409}$.

[SLIDE] Since $1,373943\dfrac{33}{64} > \left(1,172\dfrac{1}{8}\right)^2$ (another unexplained approximation by Archimedes), we have $\dfrac{OE}{AE} > \dfrac{1172\frac{1}{8}}{153}$.

[SLIDE] You probably see the pattern now. We repeat it two more times. Bisect the $7.5°$ angle to produce the $3.75°$ angle AOF as shown.

[SLIDE] Again, Euclid gives us that $\dfrac{OE}{OA} = \dfrac{EF}{AF}$.

[SLIDE] As before, this equation yields a new result.

[SLIDE] We cross multiply.

[SLIDE] Using bounds established above, we have that $\dfrac{OA}{AF} > \dfrac{2334\frac{1}{4}}{153}$.

[SLIDE] Using the Pythagorean Theorem and this inequality, we have that $\dfrac{OF^2}{AF^2} > \dfrac{5,472,132\frac{1}{16}}{23,409}$.

[SLIDE] Using another uninspired approximation, we find that $\dfrac{OF}{AF} > \dfrac{2339\frac{1}{4}}{153}$.

[SLIDE] OK, one last time! Bisect the $3.75°$ angle to create a $1.875°$ angle, AOG.

[SLIDE] Euclid gives a familiar ratio…

[SLIDE] …the same algebra give an equation…

[SLIDE] ...to which we apply cross multiplication. We then use approximations established above, to conclude that $\frac{OA}{AG} > \frac{4673\frac{1}{2}}{153}$.

[SLIDE] Now produce another 1.875° angle AOH as shown here.

[SLIDE] The central angle associated with line segment GH is $3.75° = 360°/96$. Thus GH is one side of a regular polygon of 96 sides circumscribed on the given circle.

[SLIDE] Since $AB = 2OA$ and $GH = 2AG$, it follows that $\frac{OA}{AG} = \frac{AB}{GH}$. From above $\frac{OA}{AG} > \frac{4673\frac{1}{2}}{153}$, so AB divided by the perimeter of the 96 sided polygon P is greater than $\frac{4673\frac{1}{2}}{14,688}$.

[SLIDE] So π equals the circumference of the circle divided by the diameter of the circle, which equals the perimeter of P divided by AB. We have this bounded above by $\frac{14,688}{4,673\frac{1}{2}}$ which equals $3 + \frac{667\frac{1}{2}}{4,673\frac{1}{2}}$. By reducing the denominator of the fractional part of this quantity by one, we make the fraction bigger *and* get out a "nice" quantity of $3\frac{1}{7} = \frac{22}{7}$ as an upper bound for π.

[SLIDE] With a similar technique, also starting with a 30° angle, Archimedes gets a lower bound for π of $3\frac{10}{71}$.

Archimedes and Integration

We now present an argument of Archimedes which appeared in *The Method*. Sometimes the full title of this work is used: *The Method Treating of Mechanical Problems* (such as in Encyclopedia Britannica's *Great Books of the Western World*, Volume 11). The full title is indicative of the content of this work. We will consider the proof of Proposition 1 of *The Method* and see how it relates to Archimedes' ideas of levers and balances.

[**INSERT PROPOSITION 1**] Proposition 1 of *The Method* states: "Let ABC be a segment of a parabola bounded by the straight line AC and the parabola ABC, and let D be the middle point of AC. Draw the straight line DBE parallel to the axis of the parabola and join AB, BC. Then shall the segment ABC be 4/3 of the triangle ABC."

[**INSERT ARCHIMEDES SAYS**] For this presentation, we consider a slightly simplified version of what Archimedes did by supposing that the segment AC is perpendicular to the axis of the parabola and (in our pictures) that B is the vertex of the parabola.

[**INSERT COORDINATE AXES**] Archimedes claim can easily be verified using modern calculus. We introduce coordinate axes [**INSERT FIRST SET OF AXES, INSERT SECOND SET OF AXIS**] and this allows us to find the area of the triangle [**INSERT AREA OF TRIANGLE**].

[**INSERT AREA OF PARABOLA**] With the introduced axes, we can find the equation of the parabola [**INSERT EQUATION**] and then use calculus to find the area under the parabola [**INSERT AREA**].

[**INSERT PARABOLIC SEGMENT**] We start with the parabolic segment. We introduce points and lines [**INSERT LINES; IN-**

SERT LINE; INSERT LINES].

[**INSERT MORE POINTS**] We introduce a variable point X [**INSERT POINT; INSERT LINE; INSERT POINTS**]

[**INSERT EXTENSION OF LINE AND TRANSLATION OF OX**] Now we extend line segment CK by doubling its length and placing a point T at the end, as shown. We create a line segment SH which is the same length as line segment OX and put the center of SH at point T.

[**INSERT MIDPOINT JUSTIFICATIONS**] By Proposition 33 of Book I of Apollonius' *Conics*, we have these midpoints [**INSERT APOLLONIUS**]. This also can be justified from work of Aristaeus and Euclid [**INSERT ARISTAEUS**].

[**INSERT QUADRATURE**] By Proposition 5 of Archimedes' *Quadrature of the Parabola*, $\frac{MX}{OX} = \frac{AC}{AX}$. [**INSERT PARALLEL**] Since MX is parallel to ZA, then $\frac{AC}{AX} = \frac{KC}{KN}$. [**INSERT CONCLUSION**] Therefore $\frac{MX}{OX} = \frac{KC}{KN}$.

[**INSERT MORE COMPUTATIONS**] [**INSERT FIRST**] Since $TK = KC$, we have $\frac{MX}{OX} = \frac{TK}{KN}$. [**INSERT SECOND**] Since $SH = OX$, we have $\frac{MX}{SH} = \frac{TK}{KN}$. This is the climax of Archimedes argument! He has related a "slice" of triangle AZC to a "slice" of the parabolic segment ABC. Archimedes then thinks physically. Think of the lengths of MX and SH as representing weight. We can then think of these weights as being balanced on the beam TC

with fulcrum at point K. Notice that "weight" MX is distance KN from the fulcrum, and "weight" SH is distance TK from the fulcrum, and the product of weight and distance is the same in both cases. This is justified in Book 1 of *On Plane Equilibriums*.

[**INSERT ARCHIMEDES INTEGRATES**] Archimedes views all the line segments parallel to MX inside triangle AZC as "taken together" to balance all the sections of the parabolic segment "taken together." In terms of the "taken together" concept, we have the following animation. [**INSERT ANIMATION**]

[**INSERT WHOLE AREAS**] We now must consider the centroid of the triangle. Call this centroid Y. We know from properties of triangles that Y lies 1/3 of the way from point K to point C. [**INSERT FIRST EQUATION**] That is, $KY = \frac{1}{3}KC$. [**INSERT SECOND EQUATION**] Since $TK = KC$, we have $TK = 3KY$.

[**INSERT BALANCE**] We now have the parabolic segment ABC balanced by the triangular region AZC, as seen here. This leads to the area formula:

$$\text{(area of parabolic segment)} = \frac{1}{3}(\text{area of triangle } AZC).$$

[**INSERT TRIANGLE AREAS**] Since ZA is parallel to ED and since D is the midpoint of AC, then triangle AZC is similar to triangle DEC and the lengths of the edges of AZC are twice the size of the lengths of triangle DEC. [**INSERT EQN**] Therefore $AZ = 2DE$, and since B is the midpoint of DE, we have $BD = \frac{1}{2}AZ$.

[INSERT AREAS] So:

$$(\text{area of triangle } AZC) = 4(\text{area of triangle } ABC).$$

[SLIDE: SPHERE] Archimedes used similar ideas in *On the Sphere and Cylinder* to prove that the surface area of a sphere is $4\pi r^2$ and the volume of a sphere is $4/3\pi r^3$. The volume of a sphere formula was derived by inscribing the sphere in a cylinder and showing that the volume of the sphere was 2/3 that of the volume of the circumscribing cylinder. It is rumored that Archimedes was so proud of the volume formula that he requested to have it inscribed on his tomb.

[INSERT FINAL ARCHIMEDES SLIDE] By using physical principles related to levers, Archimedes was able to perform some elementary integration 1,900 years before Newton and Leibniz independently introduced modern calculus. He is responsible for such famous formulae as the area of a circle, $A = \pi r^2$, the area of a sphere, $A = 4\pi 4^2$, and the volume of a sphere, $V = \frac{4}{3}\pi r^3$. He is, along with Euclid, one of the two most famous mathematicians of the ancient world.

[INSERT REFERENCES]

References

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3. Noel, W. and R. Netz, *Archimedes Codex, How a Medieval Prayer Book is Revealing the True Genius of Antiquity's Greatest Scientist*, Da Capo Press (2007).

4. *NOVA*, "Infinite Secrets: The Genius of Archimedes," WGBH Boston (originally aired September 30, 2003).