

PHYS 3610 Exam 2 Key

①

$$a. \Delta E = E_4 - E_3 = -7.65 \text{ eV} - (-13.6 \text{ eV}) \\ = 5.95 \text{ eV}$$

and $\Delta E = \frac{hc}{\lambda}$ for the emitted photon

$$\text{so } \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{5.95 \text{ eV}} = \underline{208 \text{ nm}}$$

b. For a hydrogenic atom

$$E_n = -\frac{13.6 Z^2}{n^2} \text{ eV}$$

$$\text{Since } E_2 = -30.6 \text{ eV} = -\frac{13.6 Z^2}{(2)^2} \text{ eV}$$

$$Z^2 = \frac{(2)^2 (-30.6 \text{ eV})}{-13.6 \text{ eV}} = 9$$

$$\underline{Z=3} \quad E_1 = \text{ground state} = -13.6(3)^2 \text{ eV}$$

$$E_1 = -122.4 \text{ eV}$$

so lowest energy photon that can ionize the atom has energy 122.4 eV

and corresponding wavelength

$$\lambda = \frac{hc}{122.4 \text{ eV}} = \frac{1240 \text{ eV} \cdot \text{nm}}{122.4 \text{ eV}} = \underline{10.1 \text{ nm}}$$

PHYS 361D Exam 2 Key

① (cont.)

c. A photon with wavelength $\lambda = 72.94 \text{ nm}$ carries energy

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{72.94 \text{ nm}} = 17.0 \text{ eV}$$

This is not enough energy to excite the electron from the ground state, and would ionize the electron from the $n \geq 3$ state. So the initial state must be $E_2 = -30.6 \text{ eV}$

$$E_{\text{final}} - E_2 = 17.0 \text{ eV}$$

so

$$E_{\text{final}} = -30.6 \text{ eV} + 17.0 \text{ eV} = -13.6 \text{ eV}$$

$$n = 3!$$

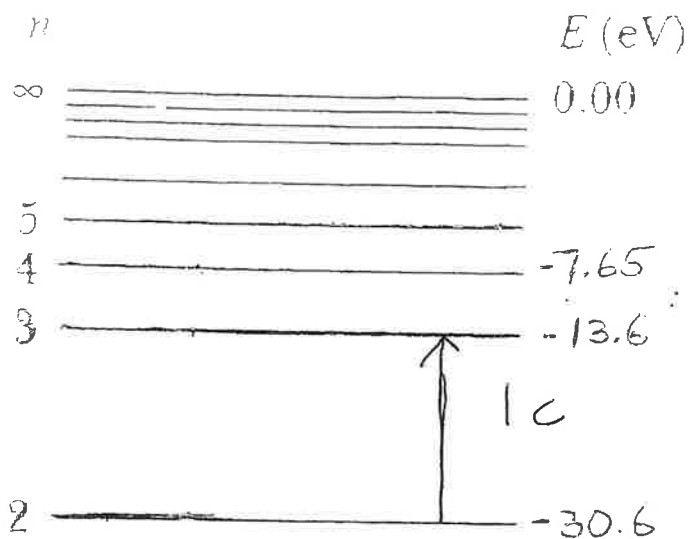
so energy diagram

Feel free to consult the sheet which contains useful constants, as well as your own equation sheets. Calculators are allowed, but no notes or books. There are 5 problems. **Answers must be clearly expressed in the appropriate units to receive full credit.** Recall $1 \text{ nm} = 1 \times 10^{-9} \text{ m}$ and $1 \text{ MeV} = 1 \times 10^6 \text{ eV}$. It may be useful to use $hc = 1.240 \times 10^3 \text{ eV}\cdot\text{nm} = 1.240 \times 10^{-3} \text{ MeV}\cdot\text{nm}$.

1. (30 points)

The accompanying figure shows the energy level diagram for a hydrogenic atom of unknown Z , as described by the Bohr model. As shown, energy labels for $n = 2$, $n = 3$, and $n = 4$ are labeled, in units of eV.

- A photon is emitted and the electron makes a transition from the third excited state to the second excited state. What is the wavelength of the photon?
- What is Z for this atom, and what is the energy and corresponding wavelength of the lowest energy photon that can fully ionize the atom from its ground state?
- Draw an arrow on the energy level diagram indicating the electronic transition associated with the **absorption** of a photon with wavelength 72.94 nm . The initial and final energy levels must be clearly indicated. You may assume that the electron remains bound. Please include your reasoning.



PHYS 3610 Exam 2 Key

2 a. $\lambda = \frac{h}{p_e}$ For electrons $K = \frac{p_e^2}{2m_e} = eV$

so $p_e = \sqrt{2m_e K}$ $K = 180. eV$

$$\lambda = \frac{hc}{p_e c} = \frac{hc}{\sqrt{2(m_e c^2) K}} = \frac{1240 eV \cdot nm}{\sqrt{2(5.11 \times 10^5 eV)(180. eV)}}$$

$$\lambda = 0.0914 nm = \text{deBroglie wavelength}$$
$$= 9.14 \times 10^{-11} m$$

b. $d \sin \theta = (1) \lambda$

so $\sin \theta = \frac{\lambda}{d} = \frac{9.14 \times 10^{-11} m}{2.40 \times 10^{-10} m}$

$$\theta = \sin^{-1}(0.3809) = 22.4^\circ \leftarrow \text{angle for first diffraction maximum}$$

PHYS 3610 Exom 2 key

③

$$\begin{aligned}\Delta p_x &= (2.3 \times 10^{-25} \text{ kg})(5.00 \times 10^{-2} \text{ m/s}) \\ &= 1.15 \times 10^{-26} \frac{\text{kg} \cdot \text{m}}{\text{s}}\end{aligned}$$

Since the HUP states

$$(\Delta x)(\Delta p_x) \geq \frac{\hbar}{2}$$

$$(\Delta x) \geq \frac{\hbar}{2\Delta p_x} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{2(1.15 \times 10^{-26} \frac{\text{kg} \cdot \text{m}}{\text{s}})}$$

$$\underline{\Delta x \geq 4.6 \times 10^{-9} \text{ m}} \leftarrow \begin{array}{l} \text{minimum uncertainty} \\ \text{in position} \end{array}$$

PHYS 3610 Exam 2 Key

④ The general form for the wave function is $\psi(x) = A \sin(kx)$

$$\text{so } k = \text{wave number} = 2.00 \times 10^7 \text{ m}^{-1}$$

The linear momentum is $\hbar k$

$$\begin{aligned} \text{so } p &= \hbar k = (1.055 \times 10^{-34} \text{ J}\cdot\text{s})(2.00 \times 10^7 \text{ m}^{-1}) \\ &= \underline{2.11 \times 10^{-27} \frac{\text{kg}\cdot\text{m}}{\text{s}}} = 3.95 \frac{\text{eV}}{c} \end{aligned}$$

The exact value for the linear momentum comes at the price of not being able to say anything about the location of the electron

It is just as likely to be in any similar interval on the x-axis

PHYS 3610 Exam 2 key

⑤ Set up a ratio

$$\frac{\Delta n(\phi = 20.0^\circ)}{\Delta n(\phi = 30.0^\circ)} = \frac{\sin^4\left(\frac{30.0^\circ}{2}\right)}{\sin^4\left(\frac{20.0^\circ}{2}\right)} \times \text{terms that cancel to unity}$$

$$\underline{\text{so}} \quad \Delta n(\phi = 30.0^\circ) = \frac{\sin^4\left(\frac{20.0^\circ}{2}\right)}{\sin^4\left(\frac{30.0^\circ}{2}\right)} \Delta n(\phi = 20.0^\circ)$$

$$\begin{aligned} \Delta n(\phi = 30.0^\circ) &= 0.2026 \left(500. \frac{\alpha}{\text{min}}\right) \\ &= \underline{\underline{101 \frac{\alpha \text{ particles}}{\text{min}}}} \end{aligned}$$