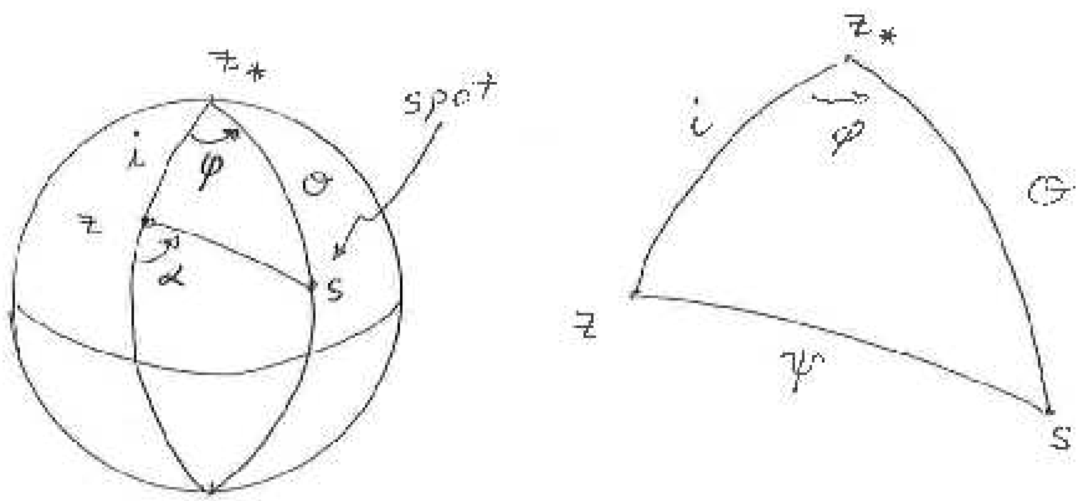
Astrophysics (Astr 3415)
Fall 2016
Prof Richard Ignace

HOMEWORK #1

1. Problem 1.8 of the text.

2. Consider a star that is spherical. The star has a spot on it. The star has spherical coordinates (ϑ, φ), and the star is rotating. The center of the spot is at a stellar co-latitude of ϑ from the star's north pole, and has stellar azimuth $\varphi = \omega t$, where t is time and ω is angular rotation speed.

Figure 1: Star geometry.



   a) Consider an observer in direction $\vartheta = i$ and $\varphi = 0$, where i is the observer's viewing inclination of this star. Both angles are fixed. The spot has coordinates of (ψ, α) as measured from the observer's z-axis (see figure). For a given inclination, prove that the spot center will NOT be occulted in its circuit around the star if it has co-latitude $\vartheta < \pi/2 - i$.

   b) Use spherical trigonometry to find a relation for the angle ψ of the spot, as seen by the observer, as a function of t, ϑ, and i.

   c) Use the expression from part (b) to determine $d\psi/dt = \dot{\psi}$ as a function of φ. Bear in mind that the star rotates like a solid body so that $\dot{\phi} = \omega$.

3. Consider a planet orbiting a distant star. If the orbit is circular, the speed will be $v(r) = v_0 \sqrt{R/r}$, for R the stellar radius and r the orbital radius. In vector form $\vec{v} = v(r)\,\hat{\varphi}$, for φ the azimuth of the orbit about the star. An observer is in the direction $(i, 0)$ along unit vector $\hat{z}$. Determine the projected line-of-sight velocity of the planet (basically the Doppler shift) in the direction of the observer as a function of r, i, φ. (This problem is somewhat similar to the previous one. Use a vector dot product. Assume the planet is in the equatorial plane of the star at a co-latitude of $\vartheta = \pi/2$.)

4. Problem 3.9 of the text.

5. For measurements with Poisson noise (such as photon counting), the "signal-to-noise" of a measurement (the quality of the measurement) is $S/N = \sqrt{S}$, because the noise $N = \sqrt{S}$.

   a) One way to increase S/N is to take multiple measurements of a source and combine them. In this case $S_{tot} = \sum_1^n S_i$, for n such measurements. The noise becomes $N_{tot}^2 = \sum_1^n N_i^2 = \sum_1^n S_i$, meaning the noise contributions add in quadrature.
   Show that if $S_0 = S_{tot}/n$ that

   $$\frac{(S/N)_{tot}}{(S/N)_0} = \frac{S_{tot}/N_{tot}}{S_0/N_0} = \sqrt{n}$$

   b) Consider a star that is a source of X-ray emission. Suppose that the X-ray photon rate $\dot{C} = 0.1$ photons/second as measured by an X-ray detector. The measured signal is $S = \dot{C} \times t$, for t the exposure time. How much time is required to obtain a measurement with $S/N = 12$?

   c) From part (b), how many photons will have been collected?

   d) In many cases measurements of astronomical sources also involve background noise. Let S be the source signal, B the background signal, and N the total noise. Suppose you measure source and background together with the instrument. The total signal is $s_{\text{tot}} = (S + B)$; the background signal B is measured separately off the source. Subtraction then gives you the source signal. But what is the noise for the source signal? (Remember: The background comes with background noise.) Formulate an expression for the S/N for the source alone.

6. Problem 5.1 of the text.

7. Problem 5.15 of the text.

8. Problem 6.7 of the text.