Astrophysics (Astr 3415)
Fall 2016
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HOMEWORK #6

1. Problem 10.3 of the text.
2. Problem 10.13 of the text.
3. Problem 10.21 of the text.
4. Problem 13.7 of the text.
5. Problem 13.8 of the text.
6. Consider a population of stars spread through space. The stars obey a "luminosity function", $\psi(L)$, for L the luminosity of the star, and ψ the relative number of stars with a given value of L. Suppose the function has the form:

$$\psi(L) = \psi_0 \, (L/L_0)^{-\alpha},$$

where L_0 and α are constants.

a) Suppose L_0 is the maximum luminosity of a star. Let L_l be the lowest luminosity of a star. If $\int \psi(L) dL = 1$, determine the constant ψ_0 in terms of L_l and L_0.

b) The apparent brightness of a star is called its flux, f, where

$$f = \frac{L}{4\pi r^2}$$

with r the distance to the star. Astronomers measure flux and often conduct surveys in terms of flux. A star of known L for a given measured f will be at distance $r = \sqrt{L/4\pi f}$.
Suppose the number density of stars is constant throughout space with n_0. However, the distribution of stars in luminosity is $\psi(L)$, with $dn/dL = n_0 \psi(L)$.
The number of stars in the interval of r to $r + dr$ is

$$\frac{dN}{dr} = \int_{L_l}^{L_0} 4\pi r^2 \, \frac{dn}{dL} \, dL$$

Derive dN/dr as a function of r by solving the integration.

c) At a given radius, stars of a different luminosity have a different flux. Observational surveys are flux-limited, meaning telescopes see stars down to a minimum brightness (call it f_0). This means only stars with $f \geq f_0$ can be detected.
At some distance (call it r_l), the least luminous star will be at the flux limit f_0. That means that for $r > r_l$, the lower limit to the integral expression of part (b) is not L_l but instead $L(r) = 4\pi r^2 f_0$. (In other words some stars are so far away that they are too faint to be seen!)
How does this affect dN/dr? Solve the equation

$$\frac{dN}{dr} = \int_{L(r)}^{L_0} 4\pi r^2 \, n_0 \, \psi(L) \, dL.$$

where $\alpha > 1$ is assumed.

d) Make a rough sketch of dN/dr. Remember, Earth is at $r = 0$. The faint limit of the survey is f_0. Let r_0 be the greatest distance at which a star of L_0 could be detected survey (so $r_0 = \sqrt{L_0/4\pi f_0}$). So your sketch has 3 zones: $r < r_l$; $r_l \leq r \leq r_0$; and $r > r_0$. It may help to recast your answer from part (c) in terms of radius r for the these zones. For example if the three zones are 1, 2, and 3, then

$$dN/dr = \begin{cases} F_1(r) \\ F_2(r) \\ F_3(r) \end{cases}$$