Astrophysics (Astr 3415)
Fall 2016
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HOMEWORK #7

1. Problem 16.6 of the text.
2. Problem 16.10 of the text.
3. Problem 16.14 of the text.
4. Consider a spherically symmetric wind that is isothermal (i.e., constant temperature) and constant ionization. If the density of the wind is a power-law with $n(r) \propto r^{-m}$, for m some constant number, show that the resulting free-free radio spectrum is a power-law with $f_\nu \propto \nu^{\alpha(m)}$, where $\alpha(m) = 2(2m-3)/(2m-1)$. The free-free opacity at radio frequencies is $\kappa_\nu \rho \propto \nu^{-2}\, n^2$. First, assume the wind is quite optically thick. Then find $R_\nu(m)$ for the condition that an observer sees to depth $\tau_\nu \approx 1$. Finally, use $f_\nu \approx 4\pi^2\, B_\nu\, R_\nu^2$, with $B_\nu \propto \nu^2$ in the radio band. Assume the Gaunt factor g_ν is a constant. What is α when $m = 2$ and $m \gg 1$?
5. Consider an emission line from a forbidden (i.e., fine structure) transition that forms in a wind.
   a) Assume a two-level atom with lower level 1 and upper level 2. Let "r" be the ion state for an elemental species (e.g., NeIII would be $r = 2$ for neon.) The level populations involve only collisional excitation from 1 to 2, collisional de-excitation from 2 to 1, and spontaneous decay from 2 to 1. Let A_{21} be the decay rate (per second), q_{12} be the collisional excitation volume rate (volume per second), and q_{21} be the collisional de-excitation volume rate. Derive n_2/n_r using $n_r = n_1 + n_2$ and $q_{12} n_1 n_e = q_{21} n_2 n_e + A_{21} n_2$. Introduce parameters of the critical density $n_c = A_{21}/q_{21}$ and $\omega = q_{12}/q_{21}$.
   b) Determine n_2/n_r for $n_e/n_c \gg 1$ and $n_e/n_c \ll 1$.
   c) Adopting $n_e = n_0 R^2/r^2$ for the density in the wind, where R is the stellar radius and n_0 the density at the wind base, determine the critical radius where $n_c = n_e(r_c)$. This radius represents a transition from a high density region where collisional rates dominate the level population balance, to one where spontaneous decay dominates. I simply want r_c/R. If $n_0 \sim 10^{13}$ cm^{-3} and $n_c \sim 10^5$ cm^{-3}, determine the value of r_c/R.

   d) Forbidden lines are optically thin. The total flux of emisison in the line is given by a volume integral over the emissivity. Introducing a kind of ion number fraction abundance $\gamma_r = n_r/n_e$, derive an expression for γ_r in terms of observed total line flux f_l using the following volume emissivity [ergs/sec/cm^2/sr]:

   $$j = \frac{1}{4\pi}\, n_2 A_{21} h\nu$$

   and line flux [ergs/sec/cm^2]

   $$f_l = \frac{1}{D^2} \int_R^\infty 4\pi j\, r^2\, dr$$

   where D is the distance to the star. The parameter γ_r is a constant (although n_r and n_e are functions of radius, their ratio is constant everywhere).

   (Note, it may be helpful to use a change of variable $u = R/r$ to evaluate the integral.)

6. a) Consider the steady-state universe model. Assume that the current density of matter is 30% of today's critical density, and that the universe is flat. So, $\kappa = 0$, $\rho_0 = 0.3\rho_c$, and $\Lambda = 4\pi G\rho_0$. Find the value of Λ.

   b) For a universe with positive curvature, and adopting the values of ρ_0 and Λ from part (a), determine the radius of curvature, R_0 for such a universe. Your answer should be expressed in Gpc.

   c) Consider a universe with curvature and a cosmological constant. (The matter density is zero.) Derive an expression for $\dot{a}$.

   d) From part (c), consider flat space with $\kappa = 0$. Derive an expression for the scale factor, $a(t)$, as a function of Λ. Recall that for time $t = t_0$, for "now", one must have $a(t_0) = a_0 = 1$.