Worksheet on Double Integrals
MATH 2110
November 3, 2011

1. Evaluate the following double integrals

    (a) $\int\int_R (x - 3y^2) \, dA$; $R = [0,2]x[1,2]$ (Ans: -12)

    (b) $\int\int_R \frac{4y}{x^3+2} \, dA$; R: $x = \frac{y}{2}$, $x = 2$, $y = 0$, $y = 4$ (Ans: $\frac{8}{3}(\ln 4 - \ln 2) = \frac{8}{3}\ln 2$)

2. Find the volume of the solid between the graphs of the given functions over the region bounded by the given curves in the xy-plane.

    (a) $f(x, y) = x^3 + y^2$; $g(x, y) = 0$; $y = 1$, $y = 2$, $x = y$, $x = y^2$ (Ans: ≈ 15.1)

    (b) $f(x, y) = \cos(x^2)$, $g(x, y) = 1$; $x = 0$, $x = \pi$, $y = 0$, $y = x$ (Ans: $\frac{\pi^2}{2} - \frac{1}{2}\sin(\pi^2)$)

3. Evaluate the iterated integral by changing it from type I to type II or vice versa

    (a) $\int_0^2 \int_{x^2}^4 e^{x/\sqrt{y}} \, dy \, dx$ (Ans: $\frac{16}{3}(e - 1)$)

    (b) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} \, dy \, dx$ (Ans: $\frac{1}{4}(e^8 - 1)$)

4. Evaluate using Fubini's theorem $\int_{-1}^1 \int_0^3 x \sin(y^2) \, dy \, dx$ (Ans: 0)

5. Use the properties of the double integrals and the values of the double integrals:

$$\int\int_R f(x, y) \, dA = 5 \qquad \int\int_S f(x, y) \, dA = 7 \qquad \int\int_R g(x, y) \, dA = 11$$

    to evaluate

    (a) $\int\int_R [f(x, y) - g(x, y)] \, dA$ (Ans: -6)

    (b) $\int\int_R [f(x, y) - 3f(x, y)] \, dA$ (Ans: -10)

    (c) $\int\int_{R\cup S} 7f(x, y) \, dA$ (Ans: 84)

    (d) $\int\int_{R\cup S}[f(x, y) + g(x, y)] \, dA - \int\int_S g(x, y) \, dA$ (Ans: 5)