MATH 2110

Test # 3

October 27, 2011

Name:____

You must **show all work** to receive full credit.

1. (6 points) Sketch the level curves of the function
$$f(x,y) = -2y + 3x^2$$
for $k = 0, 1, 2$.

2. (12 points) Find the gradient of the function
$$y^5 + x^2y^3 = x + ye^{x^2}$$
implied by the level curve, and then show that it is perpendicular to the tangent line to the curve at the point $(0, -1)$.

3. Given the function
$$f(x,y) = xe^y$$
   (a) (10 points) find the directional derivative of $f(x,y)$ in the direction of the vector $\vec{v} =< -\frac{3}{2}, 2 >$ at the point $(2, 0)$.

   (b) (6 points) find the direction of fastest increase of $f(x,y)$ at the point $(2, 0)$, and then find the rate of change of the function when it is changing the fastest at the point (2,0).

4. (12 points) Find the local extrema and saddle points of the function
$$f(x,y) = x^2 + y^2 + x^2y + 4$$

5. (12 points) Use the method of Lagrange Multipliers to find the extrema of the function
$$f(x,y) = x^2y$$
subject to the constraint $x^2 + 2y^2 = 6$.

6. (12 points) A rectangular box with a square bottom and an open top is to made from 12m^3 of cardboard. Use the method of Lagrange multipliers to determine what dimensions for the box yield the maximum volume?

7. (8 points) Write the linear transformation
$$T(u,v) =< 2u + 3v, 3u - 2v >$$
in matrix form, then find the image in the xy-plane of the unit square in the uv-plane under the given transformation.

8. (10 points) Find the velocity vector in the uv-plane to the curve
$$u = t, \ \ v = -2t.$$
Then find the Jacobian matrix and the tangent vector at the point $t = 1$ to the image of the curve in the xy-plane where
$$T(u,v) =< u^2 + v^2, 4uv^2 >$$

9. (12 points) Find the Jacobian determinant and area differential for the transformation
$$T(u,v) =< 4u^2 - v^3, u^3 - 4uv^2 >$$