MATH 2110

Test # 4

November 17, 2011

Name:____

You must **show all work** to receive full credit. All problems are 10 points each - BONUS problem on back.

1. Find the volume of the solid bound between the graphs of the functions $f(x,y) = 0$ and $g(x,y) = 2x + y$ over
$$R: \begin{array}{ll} x = 0 & y = 0 \\ x = 2 & y = 2x \end{array}$$

2. Evaluate the iterated integral
$$\int_0^{\pi/2} \int_0^x \cos x \, dydx$$

3. Evaluate the following iterated integral by changing it from type I to a type II:
$$\int_0^1 \int_x^1 \frac{\sin\left(\frac{\pi y}{2}\right) - 1}{y} \, dydx$$

4. Find the mass and center of mass of the lamina which occupies the region $[0,2]$x$[0,3]$ with mass density function $\mu(x,y) = y$.

5. Use the coordinate transformation $T(u,v) =< u, u^2 v >$ to evaluate
$$\int_1^2 \int_0^{x^2} \frac{1}{x^2 + y} \, dydx$$

6. Evaluate by converting to polar coordinates
$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{1}{\sqrt{x^2 + y^2}} \, dydx$$

7. Find the polar function $r = f(\theta)$ represented by the Cartesian equation
$$y = 2x - 1$$

8. Find the velocity vector $\mathbf{v}(\theta)$ at $\theta = \pi/4$ for the polar function
$$r = \cos(2\theta)$$

9. Find and describe the image of $S = [0,1]$x$[0,1]$ under the transformation
$$T(u,v) =< u - v, uv > .$$
Then compute the area of the image.

10. Suppose $\rho(x,y,z) = xz(1-y)$ coulombs per cubic meter is the charge density of a "charge cloud"' contained in the "box"' given by [0,1]x[0,1]x[0,1]. What is the total charge inside the box?

BONUS (3 points) The manager of a movie theater determines that the average time moviegoers wait in line to buy a ticket for this weeks's film is 10 minutes and the average time they wait to buy popcorn is 5 minutes. If X and Y are the random variables for "waiting in line to buy a ticket" and "waiting in line to buy popcorn", respectively, then their probability density functions are given by

$$p_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{10}e^{-x/10} & \text{if } x \geq 0 \end{cases} \quad p_2(y) = \begin{cases} 0 & \text{if } y < 0 \\ \frac{1}{5}e^{-y/5} & \text{if } y \geq 0 \end{cases}$$

Assume the waiting times are independent. Set up the equation describing the probability that a moviegoer waits a total of less than 20 minutes before taking his or her seat. DO NOT EVALUATE - ONLY SET UP.