MATH 2110

Sections 4.7, 5.1-5.4

December 7, 2011

1. Convert to cylindrical coordinates and evaluate.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{0}\int_{0}^{1}\frac{z}{x^2+y^2+1}\,dz\,dy\,dx$$

Ans: $\frac{\pi}{4}\ln 2$

2. Evaluate the following triple integral using spherical coordinates.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}}\frac{y}{x^2+y^2+z^2}\,dz\,dy\,dx$$

Ans: 0

3. The volume charge density of a charge cloud contained in a sphere of redius 1 meter is given by

$$\rho(x,y,z)=\frac{\rho_0}{\sqrt{x^2+y^2+z^2}}\ \text{C/m}^3$$

Calculate the total charge inside the sphere where ρ_0 is assumed to be constant. Ans: $2\pi\rho_0$

4. Construct the given vector field for

$$\mathbf{F}(x,y)=<x^3-3xy,3x^2y-y^3>$$

on a unit grid $[-1,2]$x$[-1,2]$. For visualization purposes, scaled each vector by 1/4.

5. Find the gradient vector for

$$U(x,y,z)=\sin(xz)+\sin(yz)$$

Ans: $\nabla U=<z\cos(xz),z\cos(yz)>$

6. Compute the divergence and curl of the following vector field and determine whether the vector field is conservative. If the vector field is conservative, find the associate potential for the vector field.

$$\mathbf{F}(x,y)=<\cos x\cos y,-\sin x\sin y>$$

Ans: $\text{div}\mathbf{F}=-2\sin x\cos y$; $\text{curl}\mathbf{F}=<0,0,0>$; $U(x,y)=\sin x\cos y+k$

7. Find the work done by an onject moving along the given curve through the given vector field.

$$\mathbf{F}(x,y)=<x^2-y^2,2xy>;\ \ C:\mathbf{r}(t)=<e^t,e^{-t}>,t\text{ in }[0,1]$$

Ans: $\frac{1}{3}e^3+\frac{3}{e}-\frac{10}{3}$

8. Evaluate

$$\int_{(0,0,2)}^{(1,\pi,1)}yz\sin(xy)\,dx+xz\sin(xy)\,dy-\cos(xy)\,dz$$

Ans: 3

9. Use Green's theorem to evaluate the following line integral:

$$\oint_{\partial D}(x^2+y^2)\,dx+(x^2+y^2)\,dy$$

Ans: 0