# Worksheet on Section 4.3-4.5

## MATH 2110

### November 11, 2011

1. Find the mass and center of mass of the lamina with the mass density

$$\mu(x, y) = 2x \text{ kg per square meter}$$

over the region

$$y = 0,\ y = 1,\ x = y,\ x = y^2$$

Answer: $M = 2/15$; $\overline{x} = 15/28$, $\overline{y} = 15/24$

2. Find the centroid of the region $y = 0$, $y = 1$, $x = 0$, $x = \sin(\pi y)$. Answer: $\overline{x} = \pi/8$, $\overline{y} = 1/2$.

3. Show the following is a joint probability density function over the given sample space. Then find the expected values of the random variables $X$ and $Y$.

$$p(x, y) = 4xy; \quad S = [0, 1]\text{x}[0, 1]$$

Ans: $E(X) = \frac{2}{3}$; $E(Y) = \frac{2}{3}$

4. Find the describe the image of the region $S = [0, 1]\text{x}[0, 1]$ under the transformation $T(u, v) =< u - v, u + v >$. Then compute the area of the image. Ans: 2

5. Use the given transformation to evaluate the given iterated integral

   (a) $\int_1^2 \int_y^{y+1} \frac{dx\ dy}{\sqrt{xy - y^2}}$; $T(u, v) =< u + v, v >$; Ans: $4\sqrt{2} - 4$

   (b) $\int \sqrt{xy^3}\ dA$; $T(u, v) =< \frac{u}{v}, uv >$; $R$ is bounded by $xy = 1$, $xy = 9$, $y = x$, $y = 4x$; Ans: 40

6. Evaluate the following by transforming to polar coordinates.

   (a) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{x}{x^2+y^2}\ dx\ dy$; Ans: 1

   (b) $\int_0^1 \int_1^{\sqrt{2-y^2}} \frac{y}{x^2+y^2}\ dx\ dy$; Ans: $-1 + \sqrt{2} + \ln(\sqrt{2}/2)$

7. The curve

$$r = 4\cos(3\theta); \quad \theta \text{ in } [0, \frac{2\pi}{3}]$$

is a polar curve whih enclosed a region that contains the origin. Find the area of the region the curve encloses. Ans: $\frac{8\pi}{3}$