Section 4.1
Antiderivatives
MATH 1190

- Suppose $g(x)$ is a function such that $g'(x) = 8$. What do you suppose $g(x)$ is equal to?
  Guesses:
  - $g(x) = 8x$
  - $g(x) = 8x - 10$
  - $g(x) = 8x + 3$
  - $g(x) = 8x + \sqrt{3}$
- *Antidifferentiating* is the opposite of differentiating, also called *integration*.
- **Notation:**
$$\int 8\ dx = 8x + C$$
  where $\int$ is the integration sign and dx represents what variable you are integrating with respect to.
$$\boxed{\int f(x)\ dx = F(x) + C}$$
  Here $F(x)$ is called the *antiderivative* of f, i.e. $F'(x) = f(x)$.
- **Integration Formulas:**
  - **Integration of a Constant**
$$\boxed{\int k\ dx = kx + C,\ k \text{ is any constant}}$$
    Example:
$$\int \frac{1}{2}\ dx = \frac{1}{2}k + C$$
  - **Integration of x^r, $r \neq -1$**
$$\boxed{\int x^r\ dx = \frac{x^{r+1}}{r+1} + C,\ r \neq -1}$$
    Example:
$$\int x^2\ dx = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$$
    To check: $\frac{d}{dx}\left(\frac{x^3}{3} + C\right) = x^2$ (CORRECT!!)
  - **Constant Multiple Rule**
$$\boxed{\int kf(x)\ dx = k\int f(x)\ dx + C,\ k \text{ is any constant}}$$
  - **Sum and Difference Rule**
$$\boxed{\int (f(x) \pm g(x))\ dx = \int f(x)\ dx \pm \int g(x)\ dx}$$
  - **Examples**
    1. Find $\int (x^2 - x + 2)\ dx$
$$\begin{aligned}\int (x^2 - x + 2)\ dx &= \int x^2\ dx - \int x\ dx + \int 2\ dx && \text{sum and difference rule}\\ &= \frac{x^{2+1}}{2+1} - \frac{x^{1+1}}{1+1} + 2x + C && \text{using the rules above}\\ &= \frac{x^3}{3} - \frac{x^2}{2} + 2x + C && \text{simplifying}\end{aligned}$$

2. Find $\int (x^4 + \frac{1}{8\sqrt{x}} - \frac{4}{5x^{2/5}})\ dx$

$\int (x^4 + \frac{1}{8\sqrt{x}} - \frac{4}{5x^{2/5}})\ dx$	$=$	$\int (x^4 + \frac{1}{8}x^{-1/2} - \frac{4}{5}x^{-2/5})\ dx$	rewriting everything as x^r in the numerator
	$=$	$\int x^4\ dx - \frac{1}{8}\int x^{-1/2}\ dx - \frac{4}{5}\int x^{-2/5}\ dx$	Using the sum and difference rule as well as the constant multiple rule
	$=$	$\frac{x^{4+1}}{4+1} - \frac{1}{8}\frac{x^{-1/2+1}}{-1/2+1} - \frac{4}{5}\frac{x^{-2/5+1}}{-2/5+1} + C$	using the rules above
	$=$	$\frac{x^5}{5} - \frac{1}{8}\frac{x^{1/2}}{1/2} - \frac{4}{5}\frac{x^{3/5}}{3/5} + C$	simplifying
	$=$	$\frac{x^5}{5} - 2\frac{1}{8}x^{1/2} - \frac{5}{3}\cdot\frac{4}{5}x^{3/5} + C$	multiplying by reciprocal
	$=$	$\frac{x^5}{5} - \frac{1}{4}x^{1/2} - \frac{4}{3}x^{3/5} + C$	simplifying

– **Integration of x^{-1}**

$$\boxed{\int x^{-1}\ dx = \int \frac{1}{x}\ dx = \ln|x| + C}$$

Example: Find

$$\int \left(\frac{4}{x^3} + \frac{7}{x}\right)\ dx$$

$\int \left(\frac{4}{x^3} + \frac{7}{x}\right)\ dx$	$=$	$\int \left(4x^{-3} + 7\frac{1}{x}\right)\ dx$	writing in the correct form
	$=$	$4\int x^{-3}\ dx + 7\int \frac{1}{x}\ dx$	using the constant multiple rule
	$=$	$4\frac{x^{-2}}{-2} + 7\ln|x| + C$	using the integration formulas
	$=$	$\frac{-2}{x^2} + 7\ln|x| + C$	simplifying

– **Integration of e^x**

$$\boxed{\int e^x\ dx = e^x + C}$$

$$\boxed{\int e^{kx}\ dx = \frac{1}{k}e^{kx} + C}$$

Example: Find

$$\int \left(2x^5 - 4e^{3x}\right)\ dx$$

$\int \left(2x^5 - 4e^{3x}\right)\ dx$	$=$	$2\int x^5\ dx - 4\int e^{3x}\ dx$	using constant multiple and sum and difference rule
	$=$	$\frac{1}{3}x^6 - \frac{4}{3}e^{3x} + C$	using the integration formulas

– **Integration of Trig Functions**

$$\boxed{\int \cos(kx)\ dx = \frac{1}{k}\sin kx + C}$$

$$\boxed{\int \sin(kx)\ dx = -\frac{1}{k}\cos kx + C}$$

$$\boxed{\int \sec^2(kx)\ dx = \frac{1}{k}\tan kx + C}$$

$$\boxed{\int \csc^2(kx)\, dx = -\frac{1}{k}\cot kx + C}$$

$$\boxed{\int \sec(kx)\tan(kx)\, dx = \frac{1}{k}\sec(kx) + C}$$

$$\boxed{\int \csc(kx)\cot(kx)\, dx = -\frac{1}{k}\csc(kx) + C}$$

Example: Find

$$\begin{aligned} \int (2\cos(2x) - 3\sin(3x))\, dx &= 2\tfrac{1}{2}\sin(2x) - 3\tfrac{1}{3}(-\cos(3x)) + C \\ &= \sin(2x) + \cos(3x) + C \end{aligned}$$

- **Integration Resulting in Inverse Trig Functions**

$$\boxed{\int \frac{1}{\sqrt{1-k^2x^2}}\, dx = \frac{1}{k}\sin^{-1}(kx) + C}$$

$$\boxed{\int \frac{1}{1+k^2x^2}\, dx = \frac{1}{k}\tan^{-1}(kx) + C}$$

$$\boxed{\int \frac{1}{kx\sqrt{k^2x^2-1}}\, dx = \frac{1}{k}\sec^{-1} kx + C}$$

Example: Find

$$\begin{aligned} \int \left(\frac{3}{\sqrt{1-4x^2}} - \frac{2}{x}\right) dx &= 3\int \left(\frac{1}{\sqrt{1-2^2x^2}}\, dx - 2\int \frac{1}{x}\right) dx \\ &= 3\tfrac{1}{2}\sin^{-1}(2x) - 2\ln x + C \\ &= \tfrac{3}{2}\sin^{-1}(2x) - 2\ln x + C \end{aligned}$$

- **Group Work:**
  - Problems: Determine the following integrals.
    1. $\int \left(\frac{1}{7} - \frac{1}{y^{5/4}}\right)\, dx$
    2. $\int x^{-3}(x+1)\, dx$
    3. $\int \frac{4+\sqrt{t}}{t^3}\, dt$
    4. $\int 3\cos(5\theta)\, d\theta$
    5. $\int (2e^x - 3e^{-2x})\, dx$
    6. $\int \frac{1-\cos(6t)}{2}\, dt$
    7. $\int \left(\frac{2}{\sqrt{1-y^2}} - \frac{1}{y^{1/4}}\right)\, dy$
  - Answers:
    1. $\frac{1}{7}y + \frac{4}{y^{1/4}} + C$
    2. $\frac{-1}{x} - \frac{1}{2x^2} + C$
    3. $\frac{-2}{t^2} - \frac{2}{3t^{3/2}} + C$
    4. $\frac{3}{5}\sin(5\theta) + C$
    5. $2e^x + \frac{3}{2}e^{-2x}) + C$
    6. $\frac{1}{2}t - \frac{1}{12}\sin(6t) + C$
    7. $2\sin^{-1}(y) - \frac{4}{3}y^{3/4} + C$