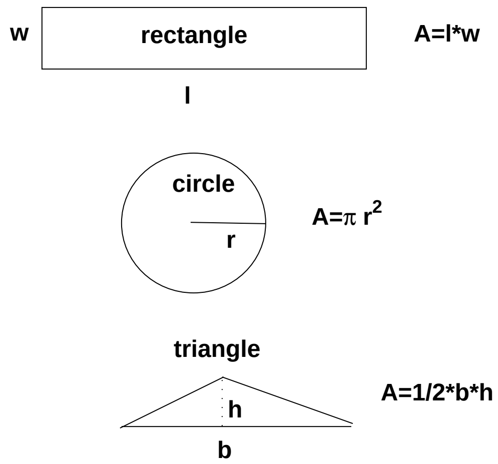
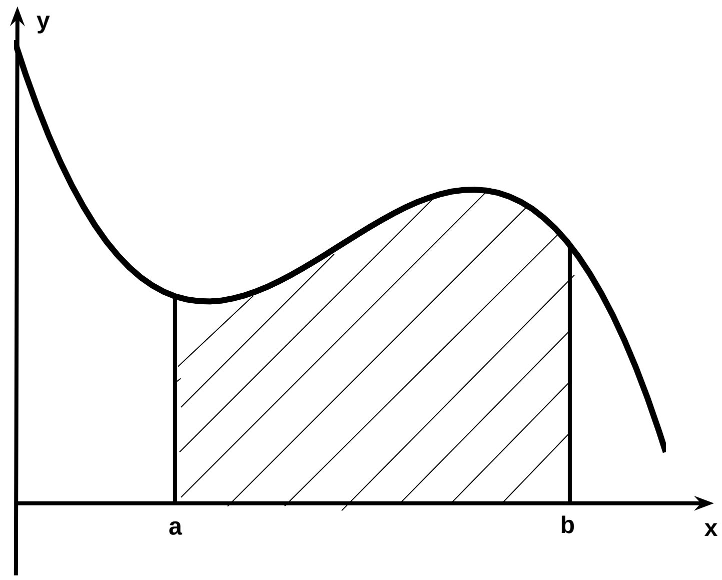
Section 4.2 to 4.4
Overview from Sections 4.2-4.4
MATH 1190

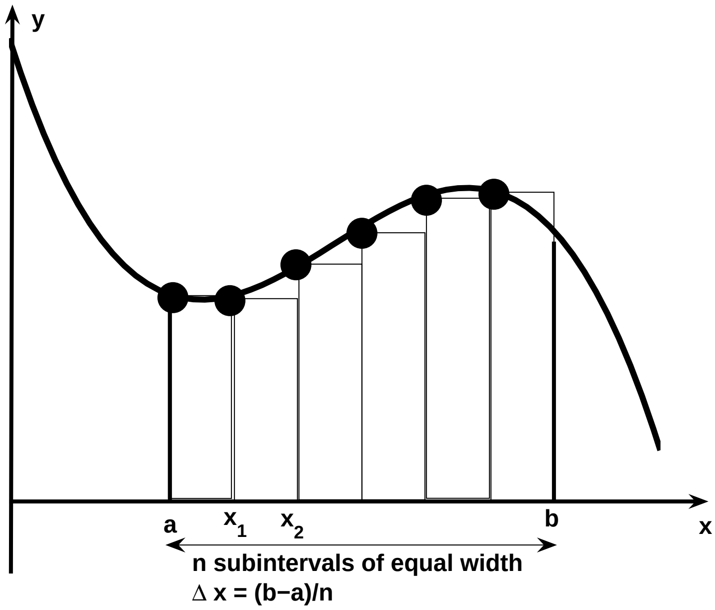
- Consider different areas.



We are looking for a different area. We are looking for the area under a curve. In certain applications the area under the curve can represent concepts such as the total profit for instance.



- You can approximate the area using rectangles.



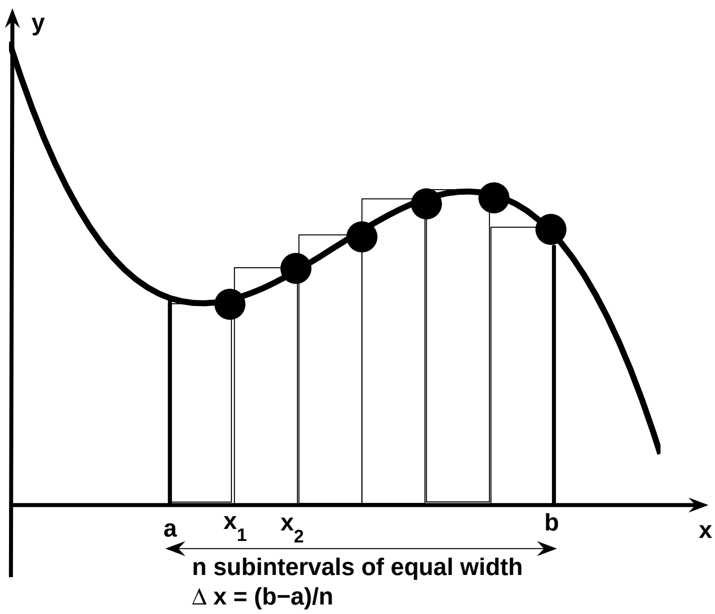
Each rectangle has width $\Delta x = \frac{b-a}{n}$ and length of $f(x_i)$. So, the area can be approximated by a sum of the areas of each rectangle

$$A \approx f(x_0)\Delta x + f(x_1)\Delta x + ... + f(x_{n-1})\Delta x$$

where $x_0 = a$. This can be written as

$$A \approx \sum_{i=0}^{n-1} f(x_i)\Delta x \text{ called the } \textbf{Riemann sum}$$

Looking at the graph, we see using the left hand endpoints probably will result in an underestimate of the actual area. We could instead use the right hand end points for the interval as shown in the following graph:



Here

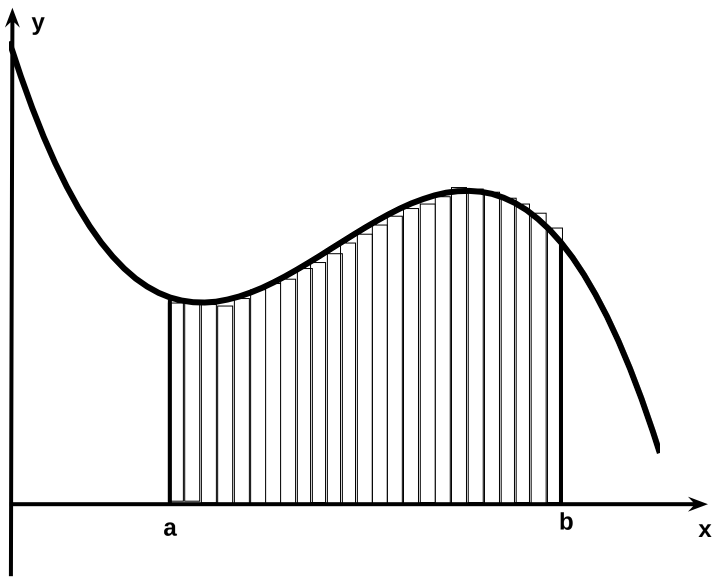
$$A \approx f(x_1)\Delta x + f(x_2)\Delta x ... + f(x_n)\Delta x$$

where $x_n = b$. This can be written as

$$A \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

This will probably lead to an overestimate of the actual area. If we add more and more rectangles, we will improve the approximate area.

- As n becomes large or as Δx becomes small we have the following picture



$$A = \begin{cases} \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\ \lim_{\Delta x\to 0} \sum_{i=1}^{n} f(x_i)\Delta x \end{cases} = \int_a^b f(x)dx$$

Here

$$\int_a^b f(x)dx$$

is called the **definite integral** with lower limit a and upper limit b.

- **Theorem:** If a function $f(x)$ is continuous on an interval $[a, b]$, then its definite integral over $[a, b]$ exists.

- **Rules for definite integrals:** We have the following rules where f and g are integrable functions.

  1. **Order of Integration**

  $$\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$$

  If you change the order of integration, the sign changes.

  2. **Zero width interval**

  $$\int_a^a f(x)\ dx = 0$$

  If the left and right hand endpoints are the same, there is no area under the graph, so the integral is 0!

3. **Constant Multiple Rule**

$$\int_a^b kf(x)\ dx = k\int_a^b f(x)\ dx$$

This is true for any k (just like indefinite integrals).

4. **Sum and Difference Rule**

$$\int_a^b (f(x) \pm g(x))\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx$$

5. **Additivity**

$$\int_a^b f(x)\ dx + \int_b^c f(x)\ dx = \int_a^c f(x)\ dx$$

where $a < b < c$.