Section 2.2
Derivative of a Function
MATH 1190

- The slope of the tangent line has a special name - the derivative at a point a. **The derivative of a function** f with respect to x is the function f' defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

- Notation for derivative:
  - $f'(x)$; "f prime of x"
  - y'; "y prime"
  - $\frac{dy}{dx}$; "dee y; dee x" or "derivative of y with respect to x"
  - $\frac{d}{dx} f(x)$; "derivative of $f(x)$ with respect to x"

- Example: Determine $f'(x)$ for $f(x) = 2x^2 + 3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Here $f(x+h) = 2(x+h)^2 + 3 = 2(x^2 + 2xh + h^2) + 3 = 2x^2 + 4xh + 2h^2 + 3$. So,

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \tfrac{2x^2+4xh+2h^2+3-(2x^2+3)}{h} \\
&= \lim_{h \to 0} \tfrac{2x^2+4xh+2h^2+3-2x^2-3}{h} \\
&= \lim_{h \to 0} \tfrac{4xh+2h^2)}{h} \\
&= \lim_{h \to 0} \tfrac{h(4x+2h))}{h} \\
&= \lim_{h \to 0} (4x + 2h) \\
&= 4x + 2(0) \\
&= 4x
\end{aligned}$$

- Example: Determine $f'(x)$ for $f(x) = \frac{1}{x}$ and use this to find the tangent line to $f(x)$ at $x = 3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Here $f(x+h) = \frac{1}{x+h}$, so

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \left(\frac{x(x+h)}{x(x+h)} \right) \\
&= \lim_{h \to 0} \frac{x - (x+h)}{h(x(x+h)} \\
&= \lim_{h \to 0} \frac{-h}{h(x(x+h)} \\
&= \lim_{h \to 0} \frac{-1}{(x(x+h)} \\
&= \lim_{h \to 0} \frac{-1}{(x(x+0)} \\
&= \frac{-1}{x^2}
\end{aligned}$$

So, $f'(x) = \frac{-1}{x^2}$, so the slope of the tangent line $f'(3) = \frac{-1}{3^2} = \frac{-1}{9}$. To find the y value, plug $x = 3$ into the original equation to get $y_1 = f(3) = \frac{1}{3}$

So, we have

$$y - y_1 = m_{tan}(x - x_1)$$

or

$$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$$

which simplifies to

$$y = -\frac{1}{9}x + \frac{2}{3}$$

- **Group Work**