

Section 2.5

Derivatives of Trig Functions

MATH 1190

- Derivative of the trig functions: MEMORIZE!!

$$\begin{aligned}
 - \frac{d}{dx} \sin x &= \cos x \\
 - \frac{d}{dx} \cos x &= -\sin x \\
 - \frac{d}{dx} \tan x &= \sec^2 x \\
 - \frac{d}{dx} \sec x &= \sec x \tan x \\
 - \frac{d}{dx} \csc x &= -\csc x \cot x \\
 - \frac{d}{dx} \cot x &= -\csc^2 x
 \end{aligned}$$

- Example: Find y' if

$$y = x^2 \cot x - \frac{1}{x^2}$$

First rewrite

$$y = x^2 \cot x - x^{-2}$$

Then using the product rule and trig derivatives on the first term and power rule on the second term, we have

$$y' = x^2 \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(x^2) + 2x^{-3} = x^2(-\csc^2 x) + \cot x(2x) + 2x^{-3} = -x^2 \csc^2 x + 2x \cot x + \frac{2}{x^3}$$

- Example: Find $\frac{ds}{dt}$ if

$$s = \frac{\sin t}{1 - \cos t}$$

Using the quotient rule, we have

$$\begin{aligned}
 s'(t) &= \frac{(1 - \cos t) \frac{d}{dt}(\sin t) - \sin t \frac{d}{dt}(1 - \cos t)}{(1 - \cos t)^2} \\
 &= \frac{(1 - \cos t)(\cos t) - \sin t \sin t}{(1 - \cos t)^2} \\
 &= \frac{\cos t - \cos^2 t - \sin^2 t}{(1 - \cos t)^2} \\
 &= \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1 - \cos t)^2} \\
 &= \frac{\cos t - 1}{(1 - \cos t)^2} \\
 &= \frac{-(1 - \cos t)}{(1 - \cos t)^2} \\
 &= \frac{-1}{1 - \cos t}
 \end{aligned}$$

- Group Work - Worksheet Part I