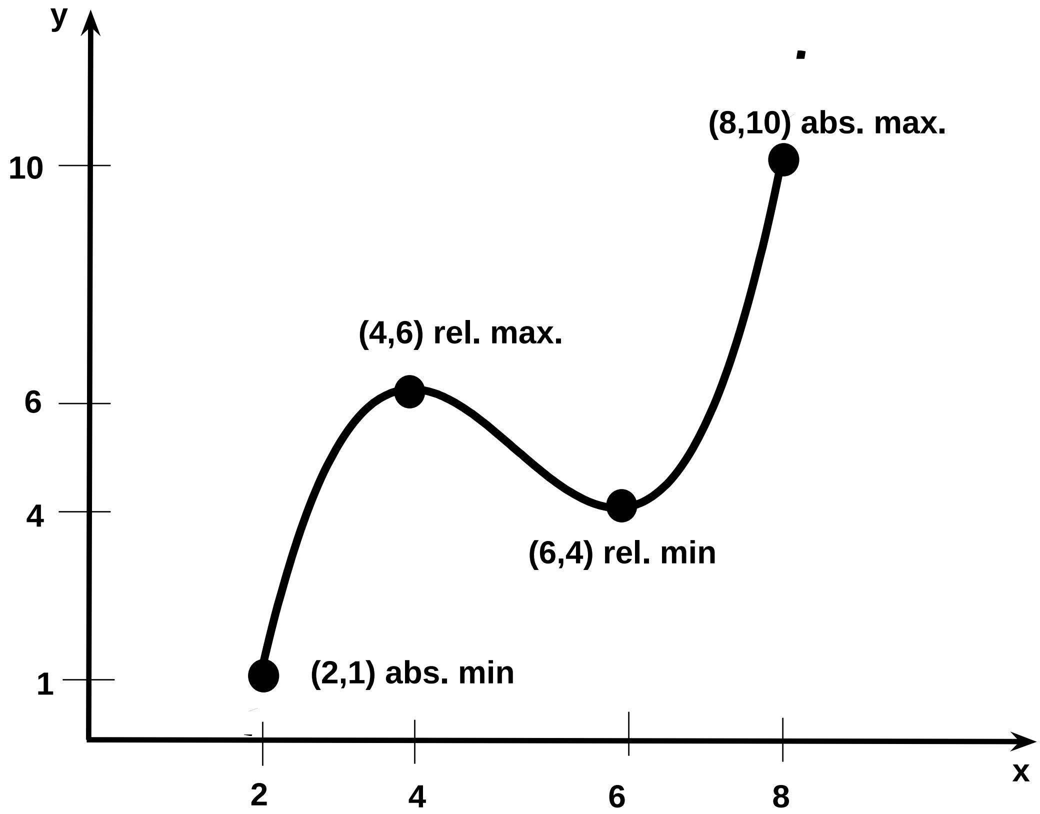
Section 3.1(part)

Extreme Values of Functions

MATH 1190



- **Relative Maximum or Minimum**: Recall that relative maximum and minimum points are maximum or minimum relative to the points close to it. They occur at critical numbers $x = c$ where $f'(c) = 0$ or $f'(c)$ is undefined. If $f''(x) < 0$ (concave down), then $x = c$ is a relative maximum. If $f''(x) > 0$ (concave up), then $x = c$ is a relative minimum.
- **Absolute Maximum or Minimum**: The *absolute* maximum or minimum is the largest or smallest value of the given interval or domain.
- For a continuous function of a closed interval $[a, b]$, the absolute maximum or minimum occur either where at one of these two points:
  1. at the relative maximum or minimum point
  2. at the endpoint of the interval
- To find the absolute extrema of the function $f(x)$ on the interval $[a, b]$:
  1. Find all critical numbers of $f(x)$.
  2. Evaluate $f(c)$ for all critical numbers *inside* the interval $[a, b]$. Evaluate $f(a)$ and $f(b)$.
  3. Find the largest value from number 2 - this gives the *absolute maximum.*
     Find the smallest value from number 2 - this gives the *absolute minimum.*
- Example: Find the absolute extrema for the function $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[-3, 0]$

First find the critical numbers: $f'(x) = 6x^2 + 6x - 12$

$$\begin{aligned} 6x^2 + 6x - 12 &= 0 \\ 6(x^2 + x - 2) &= 0 \\ 6(x+2)(x-1) &= 0 \end{aligned}$$

So, the critical numbers are $x = -2$ and $x = 1$. $x = 1$ is **not** in the interval, so disregard.

Now evaluate $f(-2)$ (original function at the critical number) and $f(-3)$, $f(0)$ (original function evaluated at the endpoints of the interval).

$$\begin{aligned} f(-2) &= 21 \leftarrow \text{abs. max at } (-2, 21) \\ f(-3) &= 10 \\ f(0) &= 1 \leftarrow \text{abs. max at } (0, 1) \end{aligned}$$

- **Group Work**
  - Problems: Find the absolute extrema of the following functions on the given interval.
    1. $f(x) = 30x - x^2$; $(-\infty, \infty)$
    2. $f(x) = x^2e^x$; $[-3, 1]$
    3. $f(x) = \frac{x}{x^2+4}$; $[0, 3)$
  - Answers:
    1. absolute maximum at $(15, 225)$
    2. absolute minimum at $(0, 0)$
       absolute maximum at $(1, e)$
    3. absolute minimum at $(0, 0)$
       absolute maximum at $(2, \frac{1}{4})$