Section 2.12
Related Rates
MATH 1190

- Recall the derivative expresses how one variable *changes* with respect to another variable. $\frac{dy}{dx}$ expresses the change of y with respect to x. This is also called a *rate of change*.
- Related rates compares one rate of change to another rate of change.
- **Example:** Suppose that the radius r and the surface area $S = 4\pi r^2$ of a sphere are differentiable functions of t. Write an equation that relates $\frac{dS}{dt}$ to $\frac{dr}{dt}$.

  To relate rates, we use implicit differentiation. In this example, both S and r are functions of time, t. So, take the derivative with respect to t on each side of the equation:

$$\begin{aligned} S &= 4\pi r^2 \\ \tfrac{d}{dt}(S) &= \tfrac{d}{dt}(4\pi r^2) \\ \tfrac{dS}{dt} &= 4\pi \tfrac{d}{dt}(r^2) \\ \tfrac{dS}{dt} &= 4\pi(2r)\tfrac{dr}{dt} \\ \tfrac{dS}{dt} &= 8\pi \tfrac{dr}{dt} \end{aligned}$$

  So, the last equation is an equation which relates $\frac{dS}{dt}$ to $\frac{dr}{dt}$.

- **Group work:**

  1. The radius r and height h of a right circular cone are related to the cone's volume V by the equation $V = \frac{1}{3}\pi r^2 h$.
     (a) How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ if r is constant?
     (b) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ if h is constant?
     (c) How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ and $\frac{dr}{dt}$ if neither r nor h is constant?
  2. If x, y, and z are lengths of the edges of a rectangular box, the common length of the box's diagonals is $s = \sqrt{x^2 + y^2 + z^2}$.
     (a) Assuming that x, y, and z are differentiable functions of t, how is $\frac{ds}{dt}$ related to $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dz}{dt}$?
     (b) How is $\frac{ds}{dt}$ related to $\frac{dy}{dt}$ and $\frac{dz}{dt}$ if x is constant?
     (c) How are $\frac{dx}{dt}$, $\frac{dy}{dt}$ and $\frac{dz}{dt}$ related if s is constant?