

# Homework #2

Math 2010

Due September 22

All problems should be worked out on your own paper showing all necessary steps required in obtaining the solution. Make sure to write down what elementary operation you are performing from step to step (you can have more than one operation for a step if desired). Make sure to state your answers clearly!! If there is an infinite set of solutions, give both the *parametric representation* for the solution and one *particular solution*.

1. Consider the system

$$\begin{array}{rccccrcr} -x_1 & + & 2x_2 & + & 5x_3 & = & -8 \\ 2x_1 & - & 2x_2 & - & 9x_3 & = & 15 \\ & & x_2 & - & 2x_3 & = & 2 \end{array}$$

- (a) Write the augmented form of the system.
- (b) Solve the system by hand using **Gaussian elimination** and back substitution. Make sure to show all necessary steps, labeling the elementary row operations performed in each step.
- (c) Continue with the row echelon form found in part (a) and solve the system using **Gauss-Jordan elimination** (i.e., obtaining the *reduced* row echelon form and using this form to solve the system). Make sure to show all necessary steps, labeling the elementary row operations performed in each step.
2. Solve the system by hand using either Gaussian elimination and back substitution or Gauss-Jordan elimination. Make sure to show all necessary steps, labeling the elementary row operations performed in each step.

(a)

$$\begin{array}{rccccccccr} -2x_1 & - & x_2 & - & 2x_3 & + & 2x_4 & - & 2x_5 & & - & x_7 & = & -5 \\ x_1 & & & + & x_3 & & & - & x_5 & & - & x_7 & = & 3 \\ 2x_1 & + & x_2 & + & 2x_3 & - & 2x_4 & + & x_5 & & + & x_7 & = & 5 \\ x_1 & - & x_2 & + & x_3 & + & 2x_4 & & & + & x_6 & - & 3x_7 & = & 6 \end{array}$$

(b)

$$\begin{array}{rccccrcr} 2x_1 & - & x_2 & + & 4x_3 & - & 3x_4 & = & 2 \\ 3x_1 & - & x_2 & + & 4x_3 & - & 2x_4 & = & 4 \\ 5x_1 & - & 2x_2 & + & 8x_3 & - & 5x_4 & = & 6 \\ & & x_2 & - & 4x_3 & + & 5x_4 & = & 3 \end{array}$$

(c)

$$\begin{array}{rccccrcr} x_1 & + & x_2 & + & x_3 & = & -1 \\ 2x_1 & - & 5x_2 & + & 4x_3 & = & 5 \\ -3x_1 & - & 2x_2 & & & = & 2 \end{array}$$

3. By hand, find values of  $a$ ,  $b$ , and  $c$  (if possible) such that the system

$$\begin{array}{rccccrcr} x & + & y & + & 2z & = & a \\ x & & & + & z & = & b \\ 2x & + & y & + & 3z & = & c \end{array}$$

- (a) has a unique solution.
- (b) has no solution.
- (c) has an infinite number of solutions.