Homework #5

Math 2010
Due October 29

Worth 20 points!!

1. (3 points each) Determine whether the given set together with the given operations is a vector space. If it is not a vector space, show that at least one property that fails to hold. If it is a vector space, you must show all 10 properties hold.

   (a) The set of all ordered triples of real numbers (x, y, z) with the operations
   $$(x, y, z) \oplus (x', y', z') = (x', y + y', z')$$
   and
   $$c \odot (x, y, z) = (cx, cy, cz).$$

   (b) The set of all ordered triples of real numbers of the form $(0, 0, z)$ with the operations
   $$(0, 0, z) \oplus (0, 0, z') = (0, 0, z + z')$$
   and
   $$c \odot (0, 0, z) = (0, 0, cz).$$

2. (2.5 points each) Determine whether the given set V is a subspace of the given set.

   (a) V is the set of all ordered pairs of real numbers (x, y) such that $x > 0$ and $y > 0$;
   $$(x, y) \oplus (x', y') = (x + x', y + y')$$
   and
   $$c \odot (x, y) = (cx, cy).$$
   Determine if V is a subspace of R^2.

   (b) V is the set of all polynomials of the form $at^2 + bt + c$ where a, b, and c are real numbers with $b = a + 1$;
   $$(a_1t^2 + b_1t + c_1) \oplus (a_2t^2 + b_2t + c_2) = (a_1 + a_2)t^2 + (b_1 + b_2)t + (c_1 + c_2)$$
   and
   $$r \odot (at^2 + bt + c) = (ra)t^2 + (rb)t + rc.$$
   Determine if V is a subspace of P_2.

3. (3 points each) Determine whether S is a basis for the indicated vector space. You must clearly show the 2 properties of basis hold or which property fails to hold if it is not a basis.

   (a) $S = \{(0, 0, 1, 1), (-1, 1, 1, 2), (1, 1, 0, 0), (2, 1, 2, 1)\}$ for R^4

   (b) $S = \{t^3 + 2t^2 + 3t, 2t^3 + 1, 6t^3 + 8t^2 + 6t + 4, t^3 + 2t^2 + t + 1\}$ for P_3

   (c) $S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ for $M_{2,2}$