Homework #6

Math 2010
Due November 17

1. Show that the matrix
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$
is diagonalizable and find the matrix P and D such that $P^{-1}AP = D$.

2. Given
$$A = \begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix}$$
   (a) Find a basis for the row space of A.
   (b) Find a basis for the column space of A.
   (c) Find a basis for the nullspace of A.
   (d) Give the nullity of A?
   (e) Give the rank of A?

3. Find a basis for the subspace of $\Re^3$ spanned by
$$S = \{[4, 4, 8], [1, 1, 2], [1, 1, 1]\}$$

4. Find a subset of vectors from the set S that is a basis for the subspace spanned by
$$S = \{[2, 7, -2, 2], [4, 14, -4, 4], [-3, -6, 1, -2], [-6, -3, -2, -2]\}$$

5. Determine whether the nonhomegeneous system $A\mathbf{x} = \mathbf{b}$ given below is consistent. If so, write the solution in the form $\mathbf{x} = \mathbf{x}_h + \mathbf{x}_p$ where $\mathbf{x}_h$ is the solution of $A\mathbf{x} = \mathbf{0}$ and $\mathbf{x}_p$ is the particular solution of $A\mathbf{x} = \mathbf{b}$. The system is given by
$$\begin{array}{rcrcrcr} x & + & 3y & + & 10z & = & 18 \\ -2x & + & 7y & + & 32z & = & 29 \\ -x & + & 3y & + & 14z & = & 12 \\ x & + & y & + & 2z & = & 8 \end{array}$$