MATH 2010

Test # 1

October 6, 2010

Name:____

You must **show all work** to receive full credit. No work = no credit!! Parts of questions will not necessarily be weighted equally.

1. (5 points each) Let $\mathbf{u} = [1, 1, 0]$ and $\mathbf{w} = [0, -\sqrt{2}, 0]$.

   (a) Find a unit vector parallel to **u**, having the same direction.

   (b) Find the angle between **u** and **w**.

   (c) Find the value of y such that $[-3, y, 10]$ is perpendicular (orthogonal) to **u**.

   (d) Find all vectors in $\Re^3$ that are simultaneously perpendicular (orthogonal) to both **u** and **w**.

2. (10 points each) Solve the system by hand using either Gaussian elimination and back substitution or Gauss-Jordan elimination.

   (a)
$$\begin{array}{rcrcrcr} 4x_1 & + & 8x_2 & + & 3x_3 & = & 11 \\ x_1 & + & 2x_2 & & & = & 5 \\ -2x_1 & - & 4x_2 & - & x_3 & = & -7 \end{array}$$

   (b)
$$\begin{array}{rcrcrcr} & & 2y & + & z & = & 4 \\ x & + & y & + & 2z & = & 6 \\ 2x & & & + & 3z & = & 9 \end{array}$$

3. (10 points) Find the values of λ so that the linear system

$$\begin{array}{rcrcl} (\lambda-1)x & - & 3y & = & 0 \\ -4x & + & (\lambda-2)y & = & 0 \end{array}$$

   will have nontrivial solutions (hints: it may help to use a determinant; make sure you know the difference between trivial and nontrivial solutions to a homogeneous system).

4. (10 points) Find x and y such that

$$\begin{bmatrix} x & y \\ 3 & 0 \\ 2x & 4 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ x & y \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 16 \\ 2 & 4 \end{bmatrix}$$

5. Given the matrix

$$A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$$

(a) (10 points) Find the determinant of A using **cofactor expansion**.

(b) (10 points) Given the answer from (a), is A invertible? If so, find A^{-1}.

(c) (5 points) Using the matrix A from above, consider the system $Ax = b$ where

$$b = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

Does the system have a unique solution? How did you determine this from the information above? If there is a unique solution, **use** A^{-1} **(found in (b))** to find the solution.

6. (1 point each) For the each of the following, determine if the statement is true or false for general matrices A and B. If it is a false statement, rewrite it so that it is true (i.e., are there special requirements on the matrices, is the multiplication or addition supposed to be performed in another order, etc.).

(a) A homogeneous system (i.e., a system in which the right hand side, b, equals 0) always has a solution.

(b) If $AB = AC$, then $B = C$.

(c) $(A^T)^T = A$ only if A is symmetric.

(d) $(AB)^{-1} = B^{-1}A^{-1}$.

(e) There is always a solution to the system $Ax = b$.

(f) $|4A| = 4|A|$ for all 4x4 matrices A.

(g) $|A + B| = |A| + |B|$ for all 5x5 matrices A.

(h) The determinant of any diagonal matrix is the product of its diagonal entries.

(i) If A is an invertible nxn matrix, then $|A^T| = |A^{-1}|$.

(j) If A is an invertible nxn matrix, then $|A^T| = \frac{1}{|A^{-1}|}$.

7. (5 points) Prove that if $A^TA = A$, **then** A **is symmetric**. Using the fact that A is symmetric (after you prove it), show that given this form $A = A^2$. Justify all your steps.