MATH 2010

Test # 2

November 1, 2010

Name:____

You must **show all work** to receive full credit. Parts of questions will not necessarily be weighted equally.

1. Given

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}$$

   (a) (5 points) Find the eigenvalues of A.

   (b) (9 points) Find the corresponding eigenvectors for the matrix.

2. (14 points) Determine whether the set of all ordered pairs of real numbers with the operations

$$(x, y) \oplus (x', y') = (y + y', x + x')$$

and

$$c \odot (x, y) = (cx, cy)$$

is a vector space. Show all supporting work for credit. If V is a vector space, you must show all properties are satisfied. If V is not a vector space, you need to only correctly show one property that fails.

Recall, a vector space V must satisfy the following properties:

(A) If $\mathbf{u}$ and $\mathbf{v}$ are any elements of V, then $\mathbf{u} \oplus \mathbf{v}$ is in V.

(A1) $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$, for $\mathbf{u}$ and $\mathbf{v}$ in V.

(A2) $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$, for $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in V.

(A3) There is an element $\mathbf{0}$ in V such that $\mathbf{u} \oplus \mathbf{0} = \mathbf{0} \oplus \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in V.

(A4) For each $\mathbf{u}$ in V, there is an element $-\mathbf{u}$ in V such that $\mathbf{u} \oplus -\mathbf{u} = \mathbf{0}$.

(S) If $\mathbf{u}$ is any element in V and c is any real number, then $c \odot \mathbf{u}$ is in V.

(S1) $c \odot (\mathbf{u} \oplus \mathbf{v}) = c \odot \mathbf{u} \oplus c \odot \mathbf{v}$ for all real numbers c and all $\mathbf{u}$ and $\mathbf{v}$ in V.

(S2) $(c + d) \odot \mathbf{u} = c \odot \mathbf{u} \oplus d \odot \mathbf{u}$ for all real numbers c and d and all $\mathbf{u}$ in V.

(S3) $c \odot (d \odot \mathbf{u}) = (cd) \odot \mathbf{u}$ for all real numbers c and d and all $\mathbf{u}$ in V.

(S4) $1 \odot \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in V.

3. (14 points) Consider the set W of all vectors in R^3 of the form (x, y, z) where $z = x - y$ with standard operations in R^3:

$$(x, y, z) \oplus (x', y', z') = (x + x', y + y', z + z')$$

and

$$c \odot (x, y, z) = (cx, cy, cz).$$

Is W a subspace of R^3? Show all supporting work for credit.

4. (a) (2 points) Explain what it means for a set S to span a vector space V.

   (b) (10 points) Determine whether the vectors $v_1 = (2, 1, -1)$, $v_2 = (-1, 3, 2)$, $v_3 = (1, 4, 1)$ and $v_4 = (5, -1, -4)$ span R^3.

5. (a) (2 points) Explain what is means to say a set S is linearly independent.
   (b) (10 points) Determine if the set $S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ is linearly independent or dependent.

6. (a) (2 points) State the requirements for a set S to be a basis for a vector space V.
   (b) (10 points) Determine whether the set $S = \{t - 1, t^2 + t - 2, t^2 + t\}$ is a basis for P_2.

7. (14 points) Express $(0, -5, 2)$ as a linear combination of the vectors $v_1 = (1, -1, 3)$, $v_2 = (2, 4, 5)$ and $v_3 = (0, 1, 1)$.

8. (1 point each) Without any work, pick the correct choice.
   (a) If a set S is in R^n has more than n vectors, S is
      i. linearly independent,
      ii. linearly dependent
      iii. not enough information is provided to determine?
   (b) If a set S is in R^n has less than n vectors, S is
      i. linearly independent,
      ii. linearly dependent
      iii. not enough information is provided to determine?
   (c) If a set S is in $M_{n,m}$ has more than $m * n$ vectors, S
      i. spans $M_{n,m}$
      ii. does not span $M_{n,m}$
      iii. not enough information is provided to determine?
   (d) If a set S is in $M_{n,m}$ has less than $m * n$ vectors, S
      i. spans $M_{n,m}$
      ii. does not span $M_{n,m}$
      iii. not enough information is provided to determine?
   (e) If a set S is in P_n has $n + 1$ vectors, S
      i. is a basis for P_n
      ii. is not a basis for P_n
      iii. not enough information is provided to determine?
   (f) If A is a nxn matrix, then $\lambda = 0$ can be an eigenvalue of A.
      i. True
      ii. False
   (g) If A is a nxn matrix, then $x = 0$ can be an eigenvector of A.
      i. True
      ii. False
   (h) If $\lambda(\lambda + 5)(\lambda - 2) = 0$ is the characteristic equation of A, then
      i. A is 2x2 and not invertible.
      ii. A is 2x2 and invertible.
      iii. A is 3x3 and not invertible.
      iv. A is 3x3 and not invertible.
      v. The size of A cannot be determined but A is invertible.
      vi. The size of A cannot be determined but A is not invertible.
      vii. Not enough information is provided to determine any of the above.