MATH 2010

Test # 3

December 3, 2010

Name:____

You must **show all work** to receive full credit. Parts of questions will not necessarily be weighted equally.

1. Given

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

   (a) (6 points) Find the geometric and algebraic multiplicity of each eigenvalue of A.

   (b) (6 points) Determine whether or not A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A and $P^{-1}AP$ *without* calculating P^{-1}.

2. Consider the basis $B = \{[2, 2], [4, 0]\}$ and $B' = \{[1, 3], [-1, -1]\}$ and assume

$$[x]_B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

   (a) (5 points) Find x in the standard basis.

   (b) (6 points) Find the transition matrix from B to B'.

   (c) (5 points) Find $[x]_{B'}$.

   (d) (6 points) If T is a linear transformation from $\Re^2 \to \Re^3$ such that $T([1, 3]) = [0, -2, 1]$ and $T([-1, -1]) = [-1, 1, 3]$, find $T(x)$ where $[x]_B$ is given as

$$[x]_B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

   Hint: use the information obtained in part (c).

3. (7 points) Find a subset of the vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ that forms a basis for the space spanned by the vectors where

$$\mathbf{v}_1 = [1, 0, 1, 1], \mathbf{v}_2 = [-3, 3, 7, 1], \mathbf{v}_3 = [-1, 3, 9, 3], \mathbf{v}_4 = [-5, 3, 5, -1]$$

4. Given

$$A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$

and it's reduced row echelon form

$$B = \begin{bmatrix} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (5 points) Find a basis for the row space of A.

(b) (5 points) Find a basis for the column space of A.

(c) (7 points) Find a basis for the nullspace of A.

(d) (1 point) What is the nullity of A?

(e) (1 point) What is the rank of A?

5. (2 points) Find the domain and codomain of the transformation $T_A(x) = Ax$ where A is a 1x6 matrix.

6. (8 points) Determine whether the linear transformation

$$T(x, y, z) = (0, 2x + 3y)$$

is linear. Show all necessary steps.

7. Given the linear transformation

$$T([x_1, x_2, x_3]) = [2x_1 - x_2 + x_2, x_2 + x_3, 0]$$

(a) (5 points) Find the standard matrix for the operator T.

(b) (5 points) Find the preimage of $\mathbf{w} = [0, -1, 0]$ for the transformation.

(c) (4 points) Determine if the transformation is one-to-one.

(d) (1 point) Does an inverse operator exist, if so, find the standard matrix for the inverse operator.

(e) (5 points) Find a basis for $ker(T)$.

(f) (5 points) Find a basis for the range of T.

8. True of False

(a) (1 point) If A is diagonalizable, then there is a unique P such that $P^{-1}AP$ is diagonal.

(b) (1 point) For every matrix A, if every eigenvalue of a matrix A has geometric multiplicity 1, then A is diagonalizable.

(c) (1 point) For every matrix A, if every eigenvalue of a matrix A has algebraic multiplicity 1, then A is diagonalizable.

(d) (1 point) Transition matrices from one basis to another are invertible.

(e) (1 point) If $ker(T) = \{0\}$, then the nullity of T is 1 and T is one-to-one.