MATH 2010

Test # 1

February 17, 2010

Name:____

You must **show all work** to receive full credit. **No work = no credit**!! Parts of questions will not necessarily be weighted equally.

1. (12 points each) Solve the system by hand using either Gaussian elimination and back substitution or Gauss-Jordan elimination. Make sure to clearly mark your solutions and show your work.

   (a)
$$\begin{array}{rcrcrcr} x_1 & - & x_2 & + & 2x_3 & = & 4 \\ x_1 & & & + & x_3 & = & 6 \\ 2x_1 & - & 3x_2 & + & 5x_3 & = & 4 \\ 3x_1 & + & 2x_2 & - & x_3 & = & 1 \end{array}$$

   (b)
$$\begin{array}{rcrcrcrcr} & & & & x_3 & + & 2x_4 & = & 3 \\ 2x_1 & + & 4x_2 & - & 2x_3 & & & = & 4 \\ 2x_1 & + & 4x_2 & - & x_3 & + & 2x_4 & = & 7 \end{array}$$

2. (12 points) How should the coefficients a, b, and c be chosen so that the system

$$\begin{array}{rcrcrcr} ax & + & by & - & 3z & = & -3 \\ -2x & - & by & + & cz & = & -1 \\ ax & + & 3y & - & cz & = & -3 \end{array}$$

   has the solution $x = 1$, $y = -1$, and $z = 2$?

3. (12 points) Find a matrix X such that

$$X \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ -3 & 1 & 5 \end{bmatrix}$$

4. (10 points) Use $|A|$ to find all the values of λ such that the system $Ax = 0$ has only the trivial solution where

$$A = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$$

5. (1 1/2 points each) Let

$$\left[\begin{array}{ccc|c} a & 0 & b & 2 \\ 0 & a & 4-b & 2 \\ 0 & 0 & b-2 & b-2 \end{array}\right]$$

   be the augmented matrix for the linear system. By inspection, for what values of a and b does the system have

   (a) a unique solution

   (b) a one-parameter solution

   (c) a two-parameter solution

   (d) no solution

6. (12 points) If possible, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$$

7. (12 points) Find $|A|$ for the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 & 0 \\ -1 & 1 & 0 & 2 \\ 0 & 2 & 0 & 3 \\ 3 & 4 & 0 & -2 \end{bmatrix}$$

8. (4 points) Using properties of matrix addition, multiplication and properties of inverses, show that if A, B, and $A+B$ are invertible matrices with the same size, then

$$A(A^{-1}+B^{-1})B(A+B)^{-1} = I$$

Make sure to justify each step with the appropriate property.

9. (1 point each) Suppose that A, B, C, D, and E are matrices with the following sizes:

$$A:(4x1) \quad B:(4x8) \quad C:(8x2) \quad D:(1x2) \quad E:(1x4)$$

Determine whether the given matrix expression is defined. For those that are defined, give the size of the resulting matrix.

(a) EA

(b) $2A+C$

(c) $C^T(EB)^T + D^T$

10. (1 point) If A is a 6x4 matrix and B is a mxn matrix such that B^TA^T is a 2x6 matrix, then what must m and n be?

11. (1 point each) Determine whether the following statements are true or false.

(a) If a linear system has more unknowns than equations, then it must have infinitely many solutions.

(b) If a homogeneous linear system has more unknowns than equations, then it must have infinitely many solutions.

(c) Adding a constant to a linear equation is an acceptable elementary row operation.

(d) If A and B are 2x2 matrices, then $AB = BA$.